THE
ENGLISH ELECTRIC
JOURNAL

VOLUME THIRTEEN • NUMBER FIVE • MARCH 1954

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All communications respecting Editorial Contents should be addressed to The Editor, THE ENGLISH ELECTRIC JOURNAL, THE ENGLISH ELECTRIC COMPANY LIMITED, STAFFORD, ENGLAND.

One of the 860 h.p. three-coach trains in service on the Lancaster-Morecambe-Heysham line of British Railways, operating from a 6,600 volts 50 cycles supply

The Lancaster-Morecambe-Heysham Line of British Railways

CONVERSION TO 50-CYCLE SINGLE-PHASE OPERATION

The first experiments with alternating current traction date back to the beginning of this century, and by 1910 the use of low frequency alternating current for traction was well established. Only a few years ago alternating current was used for nearly as many track miles as direct current.

In parallel with the development of the electrical industry, extensive three-phase electrical networks for industrial purposes spread over wide areas. It was therefore only natural that attempts were made to introduce traction making use of those existing networks, without having to convert the alternating current into direct current.

Some of the first attempts with alternating current traction arose from the existence of the 25-cycle Niagara Falls Power Supply in the U.S.A., as it was found possible to construct single-phase commutator motors to operate at this comparatively low frequency.

Similar experiments in single-phase traction were made in Europe using a frequency of 16⅔ cycles, this being a convenient frequency for conversion from the already existing 50-cycle practice.

The main attractions of applying alternating current to the overhead wire lie in the relatively high voltage that can be employed, thereby reducing the size and weight of the contact wire, catenary and supporting structures, and in being able to space substations wider apart than is possible with the relatively low voltage employed for direct current traction.

So, for instance, if for a given traffic density on a 1,500-volt D.C. system a substation were necessary every 10 miles, for the same traffic density at 3,000 volts D.C. the substations would be spaced at 20-mile intervals, whilst for 20,000 volts alternating current 50 to 60 mile intervals would suffice.

For direct current operation the substations are always converter stations with, of necessity, fairly elaborate installations. In the case of a standard-frequency alternating current supply to the contact wire the substations are merely transformers, so that with high-voltage A.C. traction the substation installation costs are obviously less for a given electrification scheme than would be the case with D.C. traction. The French claim a reduction of 40% in the cost of the fixed installation with 50-cycle traction operating at high voltage. Where, however, the frequency of the supply system differs from that used by the railway, phase and frequency conversion equipment must be provided.

The direct current series motor has been developed to a very high degree of perfection and is recognised as the most reliable motive power unit for traction.

The alternating current commutator motor, however, has a number of drawbacks. Its commutation introduces difficulties, especially during starting, and in practice it is necessary to avoid train speeds below 15% of normal maximum speed for any length of time on this account. In order to improve commutation conditions it is essential to use a large number of poles and, as a result, to operate the traction motors at a comparatively low voltage and heavy currents. This gives rise to the need for a large number of brush arms and brushes, and long commutators.

The single-phase commutator motor is certainly not a particularly simple proposition from the design point of view, and the difficulties of design increase considerably when it is decided to use a supply at a periodicity of 50 cycles, compared with 16⅔ or 25 cycles.

Comparing an equipment using 50-cycle single-phase commutator motors with one operating at a

periodicity of 16⅔ or 25 cycles, it may be said to its advantage that the transformer for the 50-cycle equipment is lighter. On the other hand, the 50-cycle motor is heavy compared with the other two on account of its low operating voltage and consequently higher current rates due to its diminished output per pole. The power factor for the 50-cycle motor is considerably lower than that for a motor operating at a periodicity of 16⅔ cycles.

In view of the inherent shortcomings of the A.C. commutator motor, it is only logical that a scheme should endeavour to apply the standard and well-proved direct current traction motor.

An attractive idea is the conversion of the single-phase supply to the locomotive into direct current for its traction motors. A simple way of doing this is by a direct current generator driven by a single-phase motor, but this has two disadvantages. The first disadvantage is that the single-phase motor driving the generator cannot be directly connected to a contact wire carrying relatively high voltage, so that a step-down transformer would have to be introduced to limit the voltage on the motor stator. The main disadvantage of this simple scheme, however, is its weight, as it must be realised that both the single-phase motor and the generator would have to handle at least the full horse-power developed by the locomotive.

In place of a motor and generator it is now quite feasible to use rectifiers and thus produce a locomotive which is not only basically very simple but also has light-weight electrical apparatus and permits the use of the best type of traction motor. The basic scheme for such a locomotive is therefore a single-phase high voltage supply, a step-down transformer carried on the locomotive, and rectifiers supplying ordinary direct current traction motors. The transformer has tappings so that the voltage supplied to the motors can be varied under the driver's control.

EXPERIMENTAL INSTALLATION

As a practical demonstration of single-phase alternating current traction operating at a periodicity of 50 cycles, it was decided to instal on the Lancaster

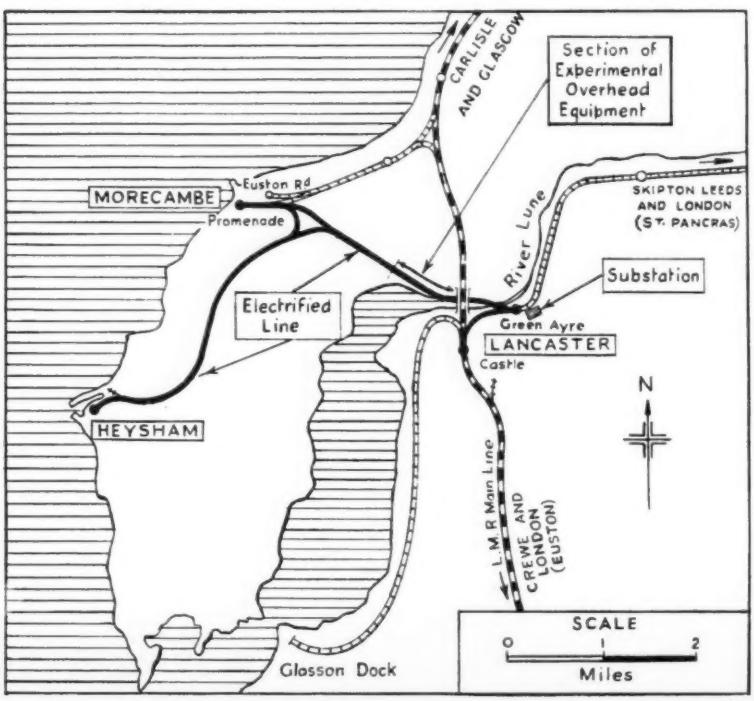


Fig. 1.—Sketch map of the Lancaster-Morecambe-Heysham electrified railway system

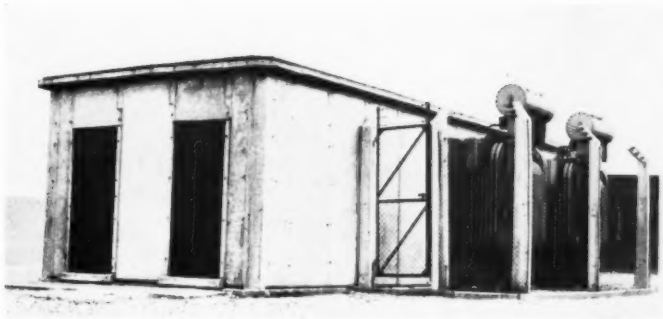


Fig. 2.—The new substation at Green Ayre, Lancaster

-Morecambe-Heysham line of British Railways three 3-coach train sets with electrical equipment designed and manufactured by The English Electric Company, together with 'English Electric' switchgear and transformers for the equipment of the substation. A sketch map of this line is reproduced in Fig. 1.

In order to provide service operation in the shortest possible time, full advantage was taken of existing industrial equipment, and much of the equipment therefore consists of items originally intended for use other than in this experimental installation. This has necessitated placing most of the equipment in a compartment above the under-frame, but the practical future development for coach stock is to have all the power equipment mounted under the floor.

Trial running was commenced in November 1952, less than 12 months after the decision to proceed with the scheme. These train sets (Frontispiece) have been operating regular passenger services since August 1953, and have thoroughly demonstrated that 'English Electric' metal-clad rectifiers are suitable for this type of duty and that, with the electric circuit employed, ordinary standard direct current traction motors give sparkless commutation under all conditions.

The Lancaster-Morecambe-Heysham line was electrified by the former Midland Railway in 1908 as an experiment to test the use of alternating current at 25 c/s for electric traction, the supply to the overhead contact wire being at 6.6 kV. The original electric trains had to be withdrawn from service in 1951 because they were worn out. In view of the recent technical progress in the design of 50-cycle train equipments it was decided to carry out trial electrification on this same railway line, as the overhead equipment was still in relatively good condition.

Substation

A new substation was built at Green Ayre station to replace the old 25-cycle motor generator plant at Heysham Harbour substation. This building, shown in Fig. 2, is a pre-cast concrete structure of standard design. The equipment supplied by The English Electric Company comprises two incoming and two transformer oil

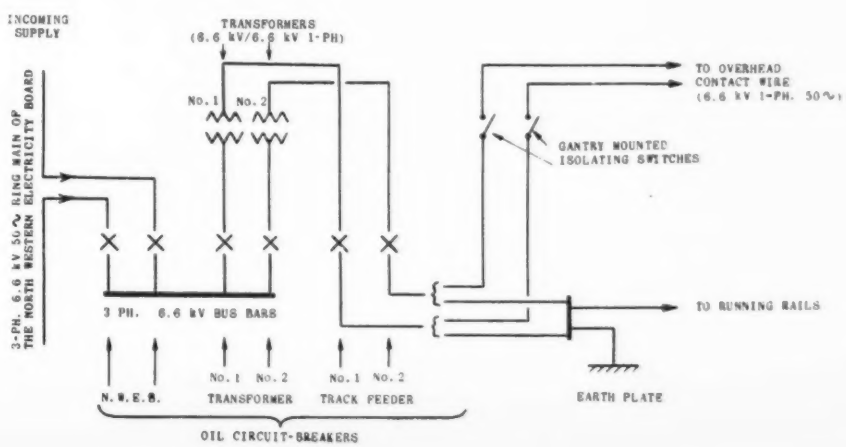

INCOMING SUPPLY
TRANSFORMERS (6.6 kV/6.6 kV 1-PH)
No. 1
No. 2
TO OVERHEAD CONTACT WIRE (6.6 kV 1-PH. 50~)
GANTRY MOUNTED ISOLATING SWITCHES
3-PH. 6.6 kV 50~ RING MAIN OF THE NORTH WESTERN ELECTRICITY BOARD
3 PH. 6.6 kV BUS BARS
TO RUNNING RAILS
N.W.E.B.
No. 1 No. 2 TRANSFORMER
No. 1 No. 2 TRACK FEEDER
EARTH PLATE
OIL CIRCUIT-BREAKERS

Fig. 3.—Diagram of H.V. connections at Green Ayre substation

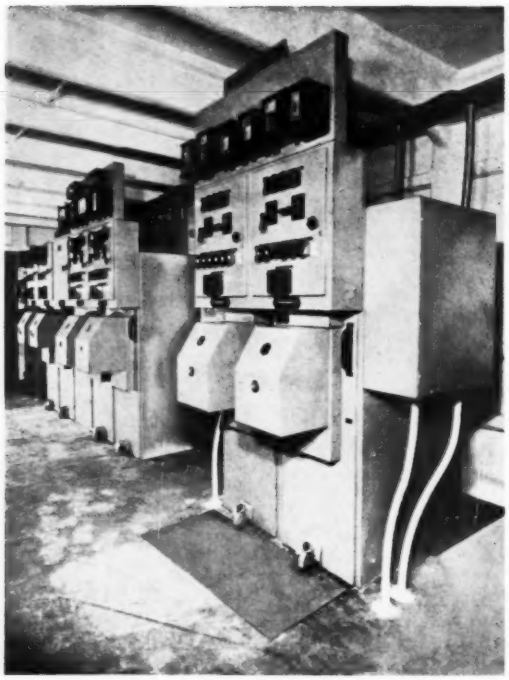


Fig. 4.—H.V. switchgear in Green Ayre substation, comprising oil circuit-breaker units for two incoming feeders, two transformers and two track feeders

circuit-breaker units, two 6.6 kV/6.6 kV 833 kVA single-phase transformers, and two track feeder oil circuit-breaker units. Fig. 3 is a diagram of the H.V. connections, and Fig. 4 shows the switchgear.

The primary of each transformer is connected between two phases of the 'grid', whilst the secondary is connected to the contact wire and to 'earth'. The transformers may be operated in parallel, but are normally used singly, one serving as standby to the other.

The track feeders have instantaneous over-current protection and the other four circuits have normal protection. The transformer and track feeder circuit-breakers are arranged for remote control from an operating panel (Fig. 5) in Green Ayre station signal box some 50 yards from the substation.

Overhead Construction

New overhead construction suitable for 20 kV was installed over about three-quarters of a mile.

The different types of supporting mast used include rolled steel joists, steel and aluminium alloy broad flange beams, lattice and tubular steel, prestressed concrete and creosoted wood. Examples of the overhead contact equipment are shown in Fig 6.

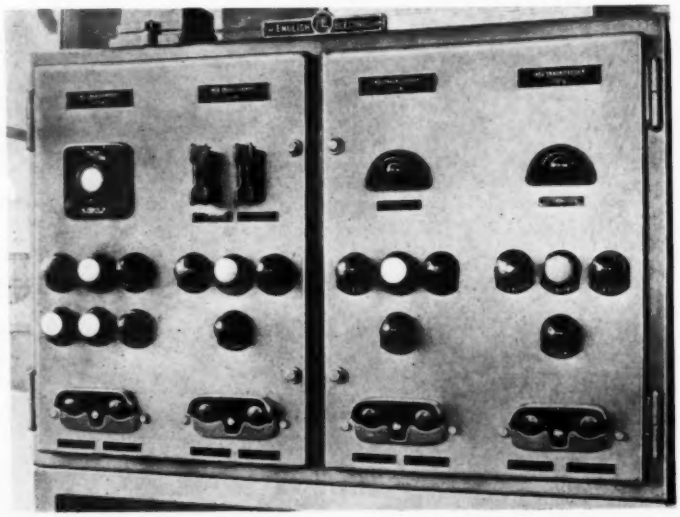
The methods used to protect the masts against corrosion include galvanising, flame cleaning and painting, pickling and painting, shot blasting and metal spraying.

Some of the hinged cantilevers are fabricated from steel angles and tees, others from steel tubes, and others from aluminium alloy sections. Pin, cap and pin, and solid core type insulators are incorporated in various cantilever arrangements.

Rolling Stock

For rolling stock, use has been made of three of the 3-coach electric train sets made surplus by the closing to passenger traffic in 1940 of the Willesden-Earls Court line in London. These train sets

Fig. 5.—Remote control panel in the signal box at Green Ayre



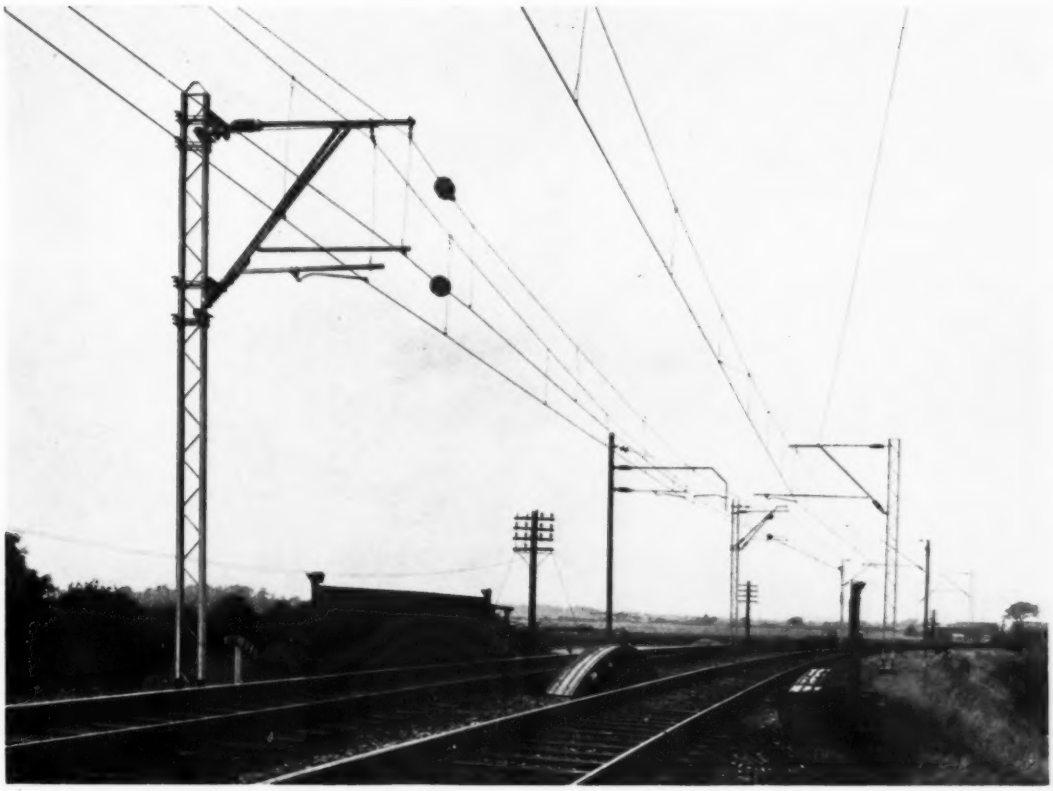


Fig. 6.—Examples of the overhead contact equipment

were built by the Metropolitan Carriage, Wagon and Finance Company in 1914 and were originally fitted with Siemens electrical equipment. Conversion of the train sets was carried out at the Wolverton Carriage and Wagon Works of the British Railways, London Midland Region.

Each train set comprises motor coach, trailer coach and driving trailer coach, of the following leading dimensions:—

Motor coach

Length over headstocks	57 ft 0 in.
Bogie centres	37 ft 6 in.
Bogie wheelbase	8 ft 9 in.
Width over body at waist ..	9 ft 0 in.
Height from rail to top of roof ..	12 ft 0 in.

Trailer and driving trailer coaches

Length over headstocks	57 ft 0 in.
Bogie centres	38 ft 0 in.
Bogie wheelbase	9 ft 0 in.
Width over body at waist ..	9 ft 0 in.
Height from rail to top of roof ..	12 ft 0 in.

Fairly extensive rebuilding of the motor coach body has been necessary to provide a larger electrical equipment compartment (Fig. 7) and a well-type flat roof for housing the pantograph, the original stock being built to operate on the 'third-rail' system. The length of the equipment compartment has been increased from 8 ft 4 in to 15 ft 0 in and the steel body framing pillars have been so disposed as to accommodate the motor operated dampers for controlling air circulation. The steel framed well roof extending over the driving and equipment compartments has been built as a complete removable unit to facilitate erection and renewal of electrical gear (Fig. 8).

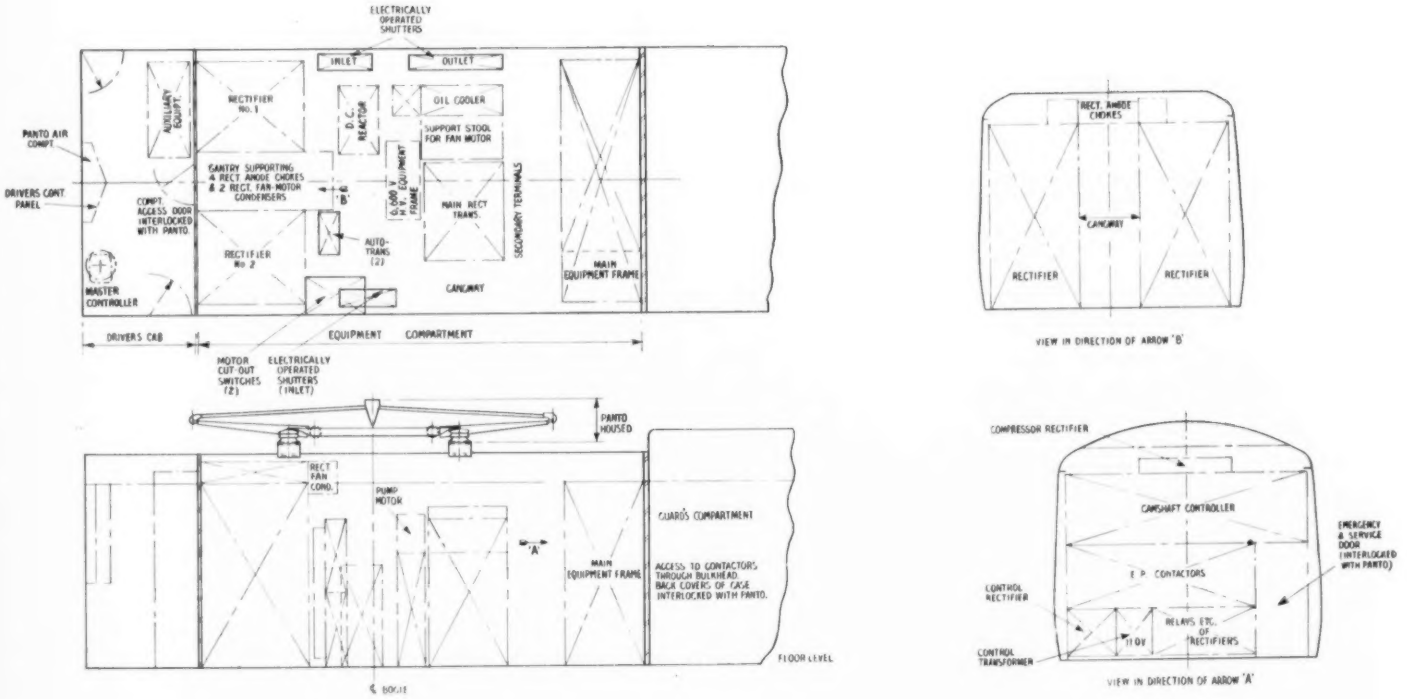


Fig. 7.—Arrangement of motor coach equipment compartment

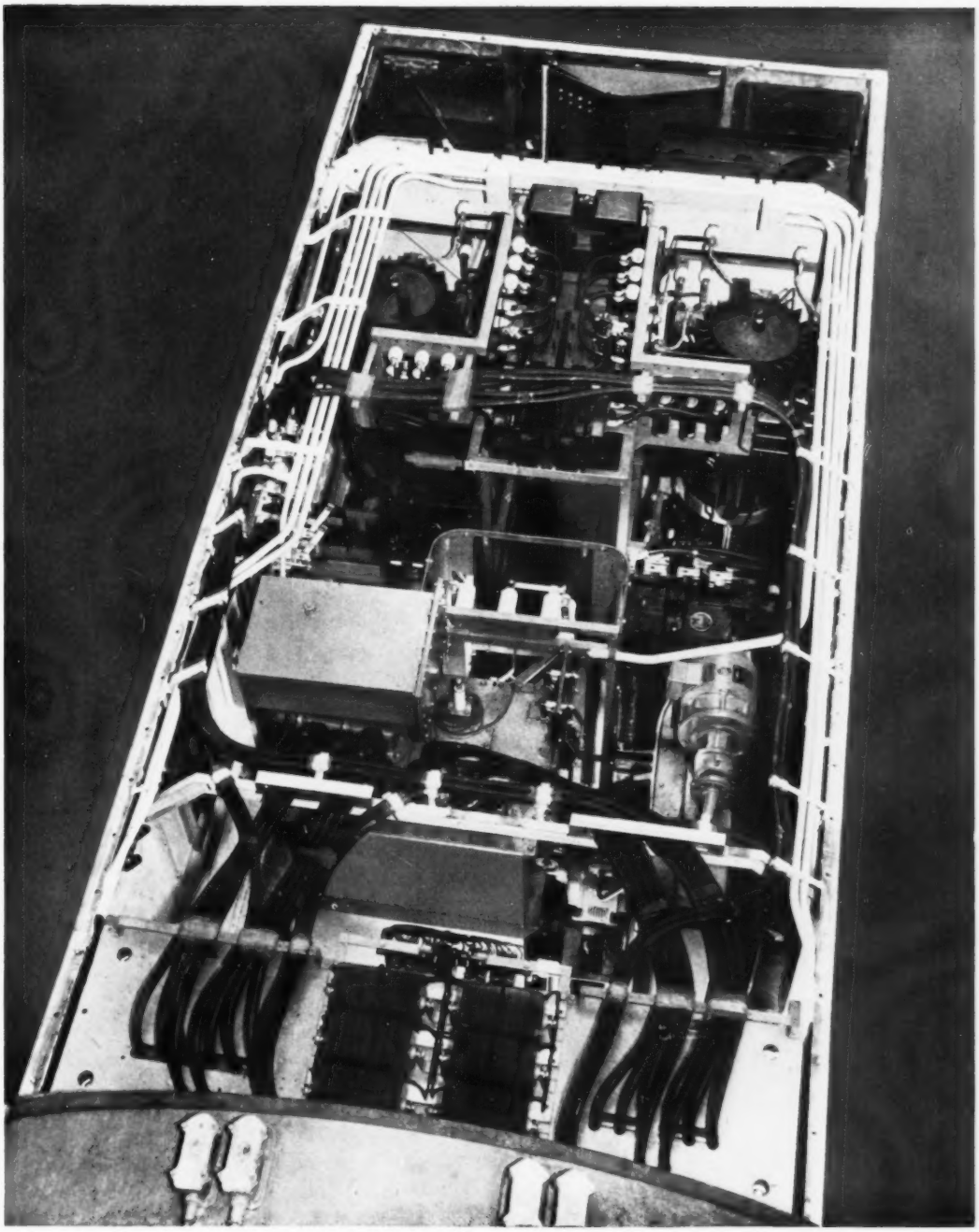


Fig. 8.—Overhead view of equipment compartment with roof removed

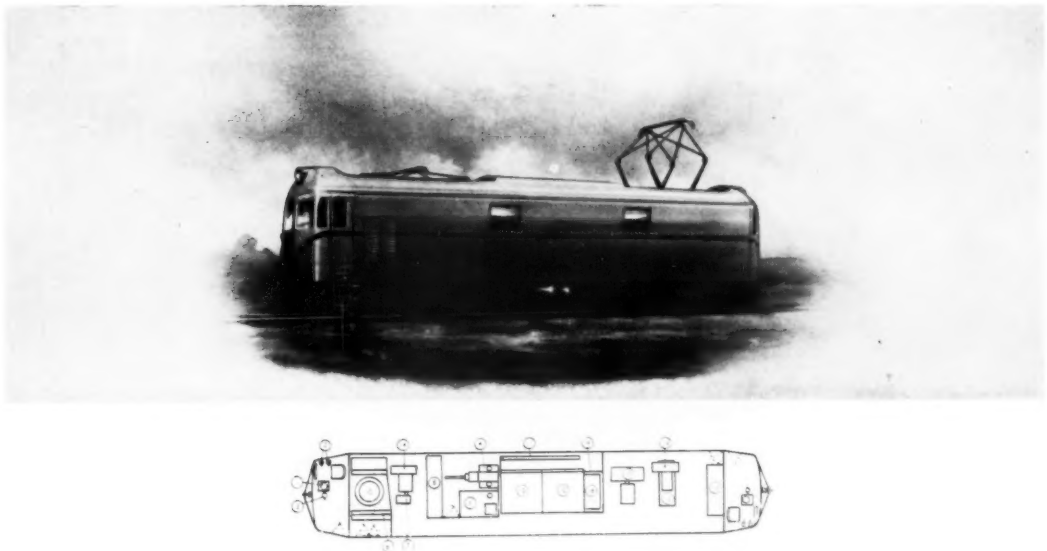


Fig. 9.—Typical rectifier operated locomotive (artist's impression)

1. Master Controller
2. Brake Valves
3. Hand Brake
4. Main Circuit Breaker
5. Main Transformer
6. Transformer Radiator
7. Transformer Oil Pump
8. A.C. Tapping Contactor Frame
9. Mid-point Auto Transformer
10. Rectifiers
11. Automatic Air Shutters
12. Smoothing Choke
13. D.C. Contactor Frame
14. Traction Motor Blowers
15. Fan and Motor for Transformer Radiator
16. Auxiliary Rectifier
17. Motor Driven Compressor

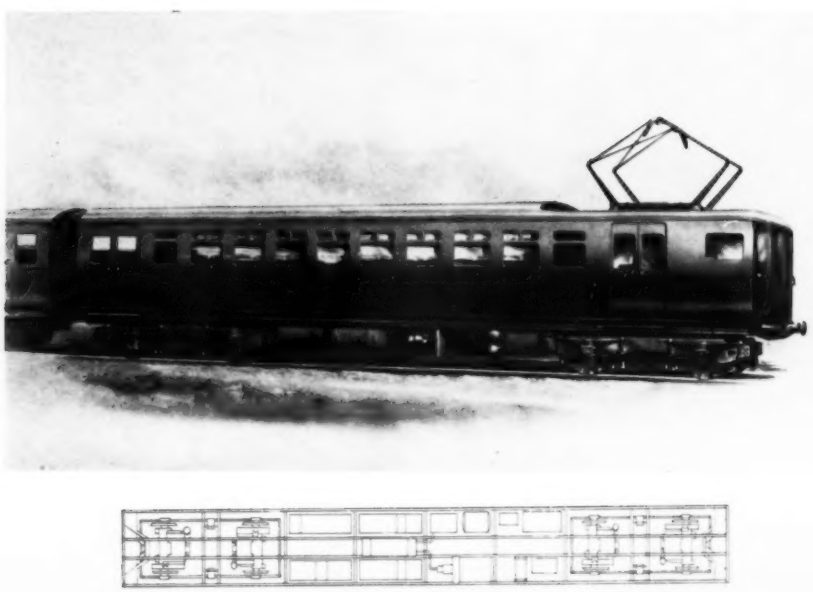


Fig. 10.—Typical rectifier operated motor-coach (artist's impression)

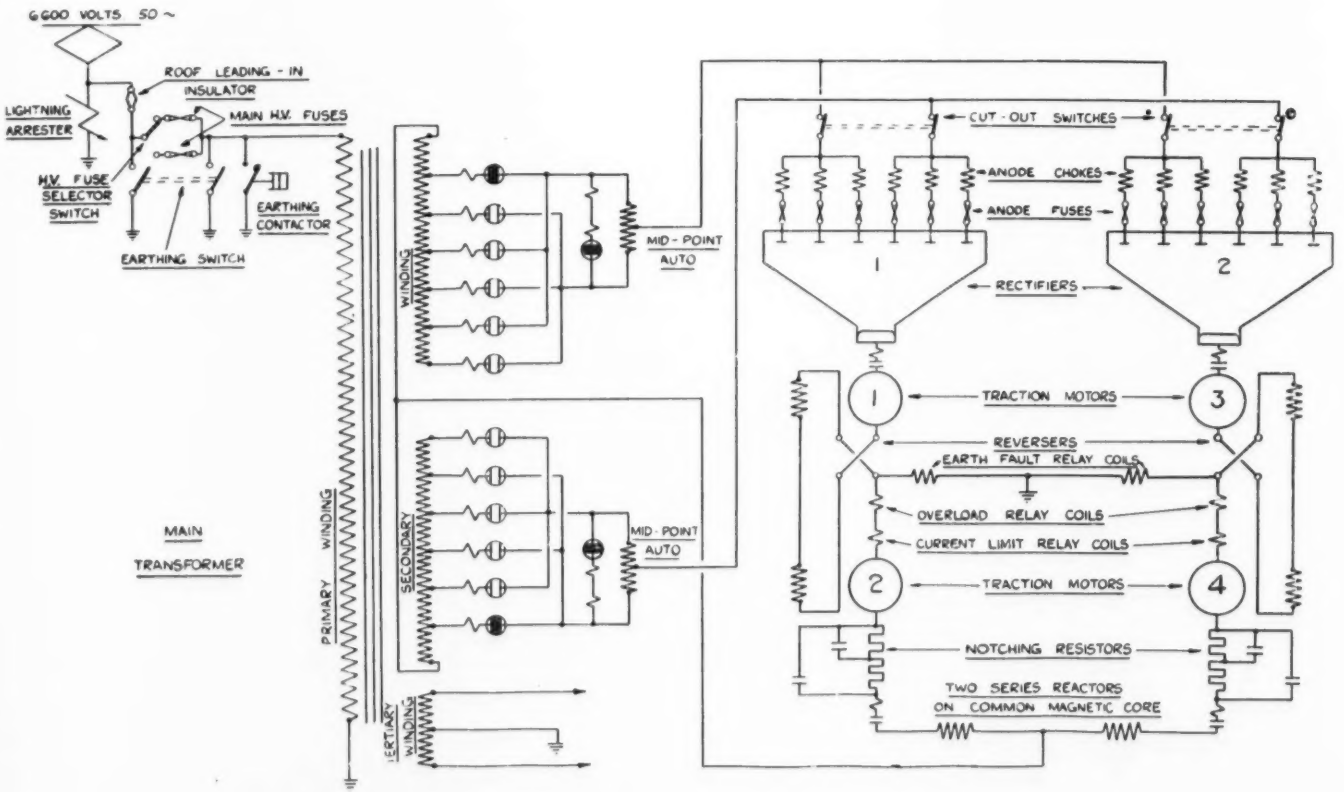


Fig. 11.—Basic diagram of power circuits for motor coach

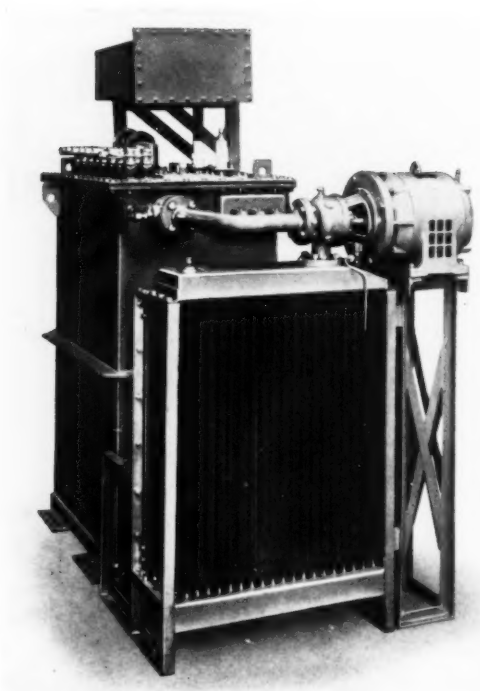


Fig. 12. (*above*)—*Rectifier transformer and oil cooling assembly*

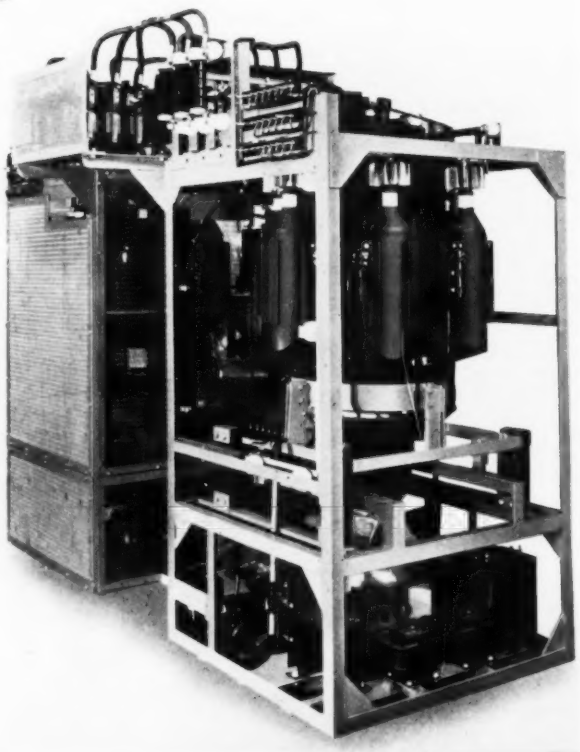
Fig. 13. (*right*)—*Rectifier cubicles before erection in motor coach*

The floor plate of the equipment compartment is of steel and the partition between this and the guard's compartment is of timber, asbestos lined on the equipment compartment side. At the top of this partition is fitted a shallow window giving a view of the well roof and enabling the pantograph to be observed in service. Separate compartments have been provided for the guard and luggage, the latter compartment being fitted with double doors.

The modifications have resulted in a reduced length of passenger compartment. All the original seats of the unit have been removed and replaced with transverse, tubular framed, coach-type double seats; details of the original and modified seating capacity are:—

Coach	Original seating	Revised seating
Motor car	40	28
Trailer car	66	62
Driving trailer car ..	60	56
Total	166	146

No structural alterations have been made to the passenger compartments of any of the coaches but the interior finish has been renovated and lightened in colour. The roof and cornice light fittings have been removed and replaced by fittings of a more modern design, and in a few instances repositioned to suit the modified seating arrangements. Most roof mouldings have been removed, flush-type ventilator under-caps fitted, and a much cleaner general appearance achieved. All interior



metalwork fittings are finished in oxidised varnished bronze.

Although the stock is old, the alterations have resulted in very comfortable and pleasant carriages. They are extremely well lit, both naturally and artificially, and the interior wood panelling is much more attractive than usual for modern suburban stock.

All underframes have been overhauled, existing conduit removed and step boards and brackets modified to suit the British Railways loading gauge. The disposition of equipment on the motor coach underframe has been entirely rearranged and has necessitated fitting numerous trimmers for mounting various details. The original brake system has been retained, but the handbrake column and associated gear originally fitted in the guard's compartment has been removed, the handbrake in the driver's compartment being retained.

Due to fitting train-line connection boxes on the ends of each coach it has been found necessary to increase the buffer projections slightly at the close-coupled ends, together with a corresponding increase in drawbar connections, to give the requisite clearance on curves.

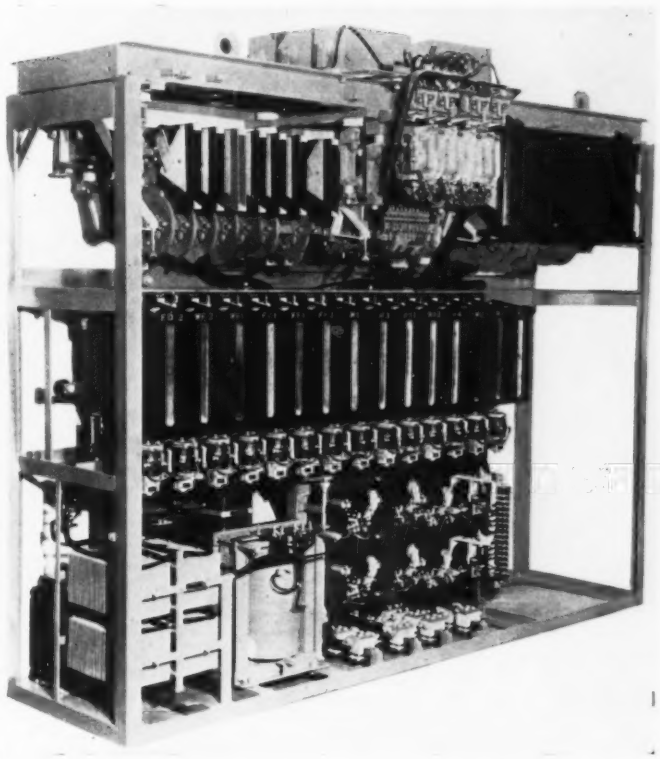


Fig. 14.—Main equipment frame, showing cam operated tap-changer, electro-pneumatic contactors and control auxiliaries

All bogies have been overhauled and slight modifications made to the motor bogie transoms and swing beams to allow for the beam-type motor nose suspension adopted for this experimental stock.

An indication of the rolling stock visualised as more typical for future rectifier operation is given by the artist's impressions in Figs. 9 and 10.

Electrical Equipment

As already mentioned, the electrical equipment of the three 3-coach train sets was supplied and erected, for experimental purposes, by The English Electric Company and, to avoid delay, equipment of existing design was used as far as possible, so that the layout is not representative of the best modern equipment.

Power at 6.6 kV, 50 cycles, is collected from the overhead contact wire by a single pantograph and fed to the equipment compartment which contains the bulk of the power equipment including transformer, rectifiers, control equipment and also part of the associated auxiliaries.

Mounted under the coach are the electro-pneumatic motor reverser, notching and motor field divert resistances, the motor-driven compressor and the 24-volt battery for emergency lighting of the equipment compartment.

The motor coach is driven by four axle-suspended self-ventilated traction motors which are D.C.

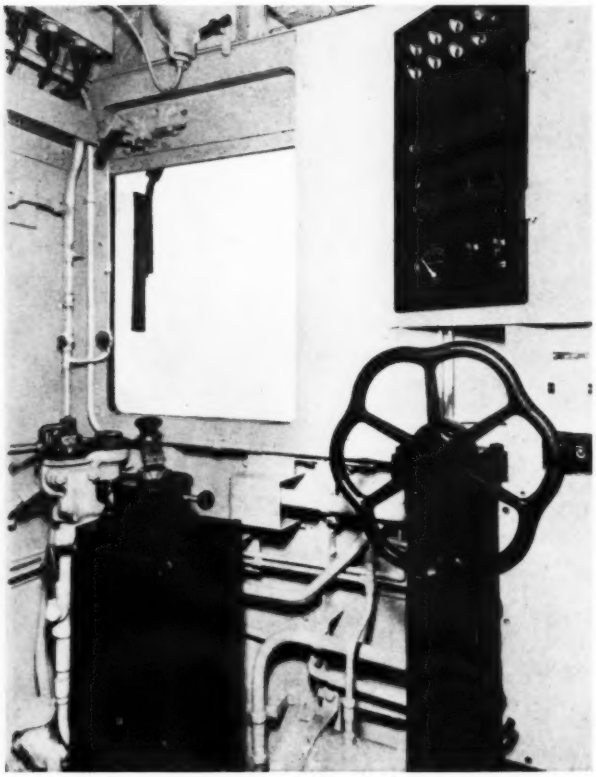


Fig. 15.—Driver's cab, showing disposition of controls and driver's control panel

series wound machines of 215 h.p. each at the one hour rating. This rating was chosen to give an equipment capable of high acceleration for experimental purposes and equal in performance to that of a normal D.C. suburban motor coach equipment; motors for this type of duty were readily available.

As shown on the power circuit diagram in Fig. 11, the high-voltage A.C. power collected by the single pantograph is fed to the traction motors via a transformer and rectifiers.

The transformer (Fig. 12) has a double secondary winding, each half of the winding being provided with tappings. It is oil cooled, and the cooling circuit includes a radiator with its motor-operated fan.

There are two rectifiers, each having six anodes which are paralleled in groups of three. The rectifiers (Fig. 13) are of the pumpless, steel bulb, mercury arc type and each is air cooled by a fan. Each rectifier feeds two traction motors connected in series, with the mid-point earthed. Some smoothing of the direct-current is obtained by the use of a D.C. reactor consisting of two series-reactors on a common magnetic core.

Control of the traction motors is obtained by varying the transformer output voltage by an 'on-load' tap-changer (Fig. 14) consisting of two sets of cam-operated contactors and auto-transformers with mid-point tappings.

The transformer contains a tertiary winding of 230-0-230 volts for auxiliary services. Train lighting is supplied at 24 volts through separate step-down transformers working off the 230-volt supply. Train heating is obtained from the tertiary winding at 460 volts. The compressor used in connection with the air brakes is driven by a standard D.C. motor supplied through a dry-plate rectifier with smoothing circuit.

There is a high-voltage fuse between the pantograph and the primary of the power transformer in the high-voltage compartment, and a lightning arrestor is provided on the roof. Overload protection is provided by overcurrent relays and contactors in the D.C. circuits, while six anode fuses are provided for each rectifier.

Temperature control of the rectifiers is provided. Each rectifier bulb is fitted with heaters controlled by a thermostat so that the bulb can be pre-heated during extremely cold weather when the equipment has been out of use for some time.

The equipment compartment is fitted with electrically operated shutters under the control of thermostats so as to control the incoming and outgoing air flow of the compartment. Thus the temperature of the air used for cooling the rectifiers is automatically controlled.

The equipments are designed so that the train sets can be operated in multiple unit.

Operation

The driver's cab (Fig. 15) contains a master controller, air-brake valve, driver's control and indication panel and a cupboard for the fuses, contactors, etc. which are used in the auxiliary and control circuits.

The pantograph is raised by residual air pressure in the pantograph reservoir, or failing this, by a foot pump, and contact with the overhead line energises the rectifier transformer through the 6.6 kV fuse.

The tertiary voltage is then available for train lighting and heating and also for the control supply, battery charger, compressor supply and auxiliaries. The compressor operates under the control of its air-controlled governor. The rectifier heaters are energised automatically. With the various control supply switches closed the driver waits for the air pressure to build up, by which time the rectifiers have warmed up.

The train is driven from the master controller in exactly the same manner as a standard D.C. equipment. The removable key is first inserted, the master switch thrown to the 'on' position, and the reverser handle to 'forward' or 'reverse'; the main power handle is then moved to the shunting, half voltage, full voltage or weak field positions. Acceleration is automatically controlled by a current limit relay up to whichever running notch is chosen.

The principal operating data are:—

Line voltage (at no load) ..	6,600 volts.
Line frequency	50 cycles/sec.
One hour rated output at wheels (allowing 10% line drop)	860 h.p.
One hour rated tractive effort (in full field)	11,500 lb.
Average accelerating tractive effort	17,900 lb.
Wheel diameter (mean) ..	42 inches.
Gear ratio	65/16.
Maximum permissible speed for equipments	75 m.p.h.

Telecommunication and Signalling Equipment

The higher current due to improved train performance and the higher frequency of supply (compared with the original 25 c/s), together with the employment of rectifiers on the motor coaches, increased the interference with existing telecommunication and signalling circuits. Tests confirmed the need for these circuits to be carried by underground cables. In addition, it was found that alterations were required to relays and to the feed-end arrangement of some 30 direct-current single rail track circuits on account of stray alternating currents in the track circuits. The relays in question had to be altered internally by providing a magnetic shunt and copper slugging so that alternating-current cannot energise them.

A Simplified Approach in Studying the Flow of the Power and Reactive Components in a Circuit

By S. E. NEWMAN, A.C.G.I., A.M.I.E.E., and A. D. CLARK, B.Sc.,
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When considering the performance of a synchronous machine connected to a system, terms such as the 'zero power factor leading (or lagging) kVA' or more briefly 'leading (or lagging) kVA' are frequently used. These terms can lead to confusion, since an over-excited synchronous condenser may be said to supply 'leading kVA', or alternatively to demand 'lagging kVA', while at the same time there is the apparent anomaly that an over-excited generator operates at a lagging power factor.

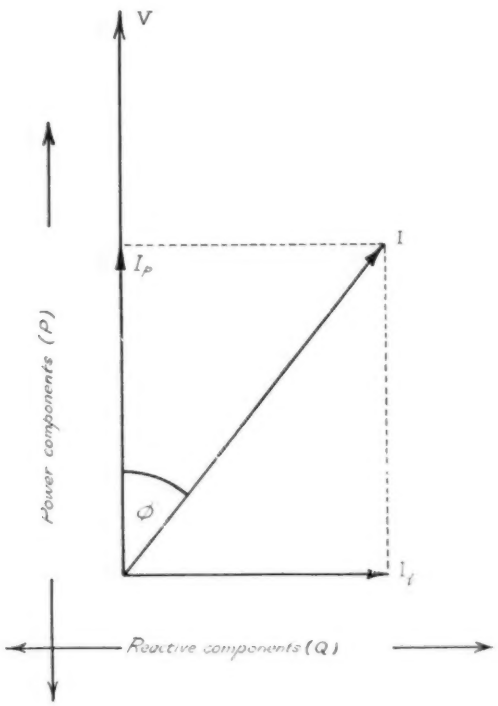


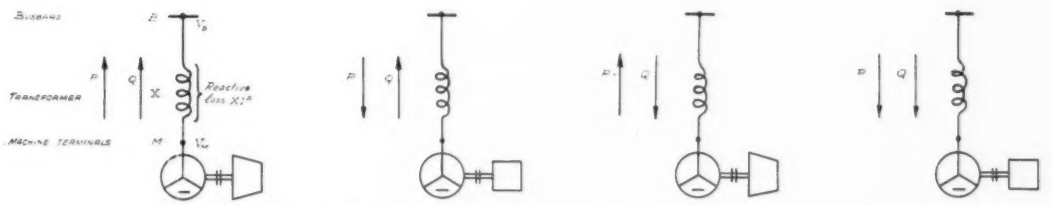
Fig. 1.—The power and reactive components of a current

In power system studies, the possibility of confusion is avoided by considering only the magnitude and direction of flow of the power and reactive components of the kVA in the system. It may not be generally realised that the same approach may be used to simplify consideration of a single machine feeding into a live busbar.

In Fig. 1, if the vectors V and I represent the phase voltage (in kilovolts) and current (in amperes) respectively in a balanced three-phase circuit, the power component P of the total kilovolt-amperes of the three-phase circuit is $3VI_p = 3VI \cos \phi$ in kilowatts, while the reactive component Q is $3VI_q = 3VI \sin \phi$ in kVAr (kilovolt-amperes reactive).

The power P and reactive Q can be considered as two separate commodities which are required and must therefore be provided in a system. Power is supplied by synchronous and asynchronous (induction) generators, and consumed by motors, resistive loads and resistive losses. Reactive is supplied by all over-excited synchronous machines (generators, condensers and motors), static capacitors and by overhead line and cable capacitances, while it is demanded by all under-excited synchronous machines, induction motors, transformers, inductive loads, and line and cable inductances.

The flow of power and reactive in a circuit is controlled by the angular displacement between the voltage vectors at the two ends and their difference in magnitude. If resistance and capacitance are neglected, and excepting cases where Q is very much smaller than P, the flow of power and reactive



(a) *Over-excited generator* (b) *Over-excited motor* (c) *Under-excited generator* (d) *Under-excited motor*

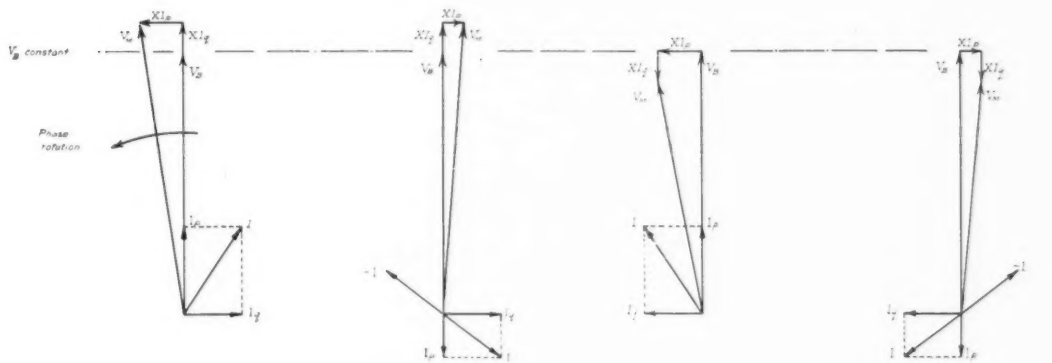


Fig. 2.—Power flow and vector diagrams for a synchronous machine and transformer combination

NOTE. – Flow of power through an inductive reactance is accompanied by a falling back of the voltage vector.
Flow of reactive through an inductive reactance is accompanied by a drop in magnitude of the voltage vector.

can be considered independently and the following general rules apply:—

(i) The direction of power flow (P) is from a point having a given voltage vector to one with a relatively lagging voltage vector.

(ii) The direction of reactive flow (Q) is from a point of higher to one of lower voltage.

(iii) When P and Q in a circuit flow in the same direction the current lags the voltage. With P and Q flowing in opposite directions the power factor is leading.

Considering now the simple case of a synchronous machine connected via a step-up transformer to busbars as in Fig. 2, then neglecting magnetisation and losses the transformer may be treated as a series reactance X connected between the machine terminals M and the busbars B. In the operating conditions considered it is assumed that other synchronous machines feed the busbars and that the busbar voltage is maintained constant.

Condition (a). Machine acting as a generator and over-excited.

As the machine is being driven by the prime mover, the flow of power is in the direction of arrow P. Similarly, as the machine is over-excited, the generator voltage V_M is higher than the busbar voltage V_B, and reactive is supplied to the system in the direction of arrow Q. It should be noted that the vector diagrams are drawn assuming, in the accepted manner, a 1 :1 transformation ratio and that the voltages V_M and V_B are expressed as percentages of their nominal values.

The flow of power through the reactance X (representing the step-up transformer) is accompanied by a lag of the busbar voltage behind that

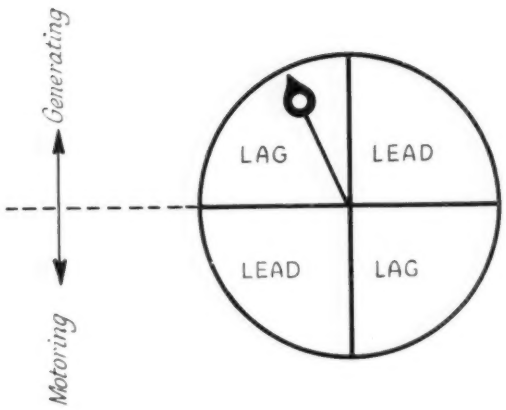


Fig. 3.—Power factor indicating instrument

of the machine. This angular change brings the voltage at the busbars closer in phase with the current, *i.e.* the power factor at the busbars is higher than at the machine. The corresponding reactive loss in the transformer is XI^2.

The power factor is lagging, indicated by the arrows P and Q lying in the same direction.

Condition (b). Machine acting as an over-excited synchronous motor.

In this case the flow of power is into the machine and thus the busbar voltage leads the machine terminal voltage. If the machine is acting as a synchronous condenser the power is merely that required to supply the losses, and consequently the phase displacement is very small.

The flow of reactive is again towards the busbars since the motor is over-excited and its voltage V_M highe than the busbar voltage V_B.

The opposite directions of the P and Q arrows indicate that the power factor is leading. In the vector diagram this can be more clearly seen if the current vector is reversed to give the current flowing into the machine (−I).

Condition (c). Machine acting as a generator and under-excited.

Since the generator is under-excited its terminal voltage V_M is lower than the busbar voltage V_B and the reactive flow is in the opposite direction to that in Condition (*a*), although the busbar voltage still lags the machine voltage.

Due to the reactive loss in the transformer the power factor at the machine terminals is in this case better than that at the busbars.

P and Q in opposite directions show that the power factor is leading.

Condition (d). Machine acting as an under-excited synchronous motor.

The flow of power is towards the machine so that the busbar voltage again leads the machine terminal voltage. The reactive flow is in the same direction as in Condition (*c*) and the voltage at the machine terminals is consequently lower than that at the busbars. If the machine is acting as an under-excited synchronous condenser, *i.e.* as a synchronous inductor, the power flow is small, supplying only the machine losses, and the vectors V_M and V_B are practically coincident.

P and Q flowing in the same direction show that the power factor is lagging. Again in the vector diagram the reversed current vector (−I) is in agreement.

Condition (e). Machine acting as a generator. Change in excitation.

Assume the generator to be operating as in Condition (*a*) and that the excitation is gradually reduced. The machine terminal voltage V_M falls, Q decreases and passes through zero so that, when V_M is less than V_B, Q is found to flow into the machine as in Condition (*c*).

No change is assumed in the driving power, and consequently P remains constant.

It may be noted that under this condition a power factor indicator on the generator would start in the upper (generating) left hand 'lag' quadrant (see Fig. 3) and gradually move into the upper right hand 'lead' quadrant.

Condition (f). Machine acting as a generator. Change in driving power.

Again assuming the generator to be operating as in Condition (*a*) and that the driving power is gradually removed; P will decrease to zero and finally there will be a small flow of power in the reverse direction to motor the machine as in Condition (*b*).

As the machine remains over-excited, the direction of flow of Q is unchanged.

In this case a power factor indicator on the generator would gradually move from the upper left hand 'lag' quadrant to the lower (motoring) left hand 'lead' quadrant.

General Application.

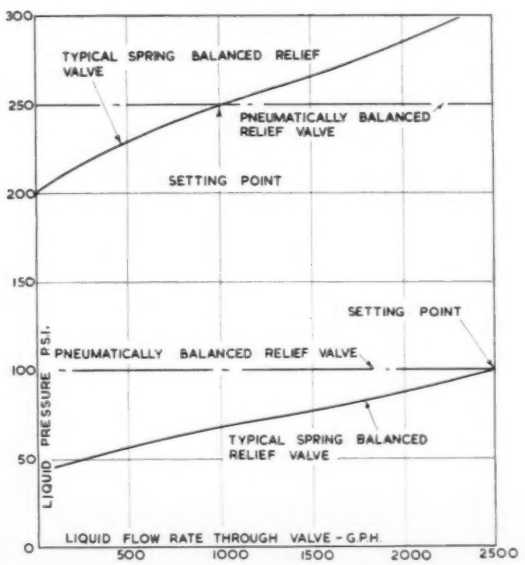
The presence of resistance can cause the general rules to be modified. However, provided that the amount of resistance in circuit is small, this simple 'P and Q' approach is of great assistance in the qualitative assessment of the performance of power circuits.

The Napier Pneumatically Balanced Relief Valve

DURING THE COURSE of a research programme recently carried out by the Luton Flight Development Establishment of D. Napier and Son Limited, it was found necessary to arrange for the rig testing of fuel and oil systems before their inclusion in an aircraft engine installation. The test rigs designed for this purpose required a relief valve that was suitable for operation at high pressures and flow rates, at the same time being easily adjustable to cover a large pressure range and yet remaining equally responsive throughout the range.

It soon became apparent that the existing types of spring-balanced relief valves were not suitable, due to their restricted range of adjustment and because the change of pressure resulting from a change of flow rate was excessive due to the spring rating. As a result of the research which ensued, the Pneumatically Balanced Relief Valve has been evolved having many advantages over the spring-type valve previously used. Not only can it handle high flow rates and pressures but also high changes of flow rates with negligible change of pressure. This is well illustrated in Fig. 1, where its performance is compared with that of a typical spring-loaded valve. The pneumatically balanced valve is easily adjustable over a considerable pressure range and can be remotely controlled whilst in operation. Other advantages are the relative ease of manufacture, as very few close tolerances are required, and absence of mechanical limits restricting the capacity for which a valve can be designed. Its

Fig. 1.
Comparison of spring and pneumatically balanced relief valve performance



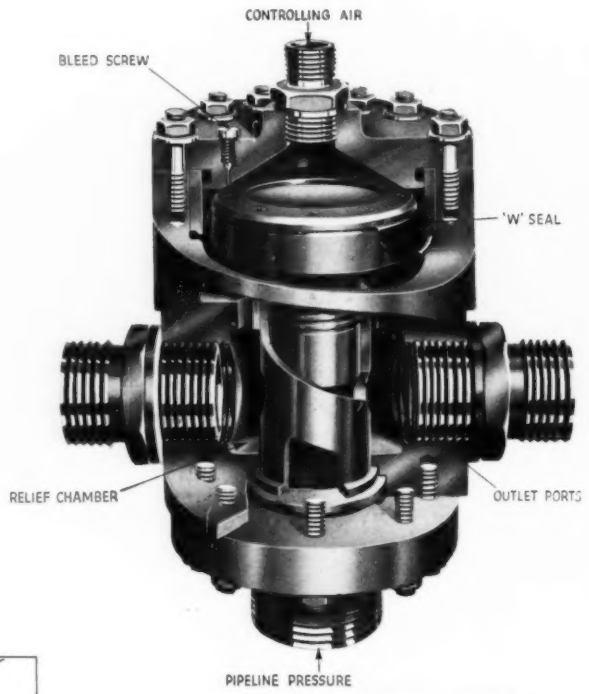


Fig. 2.—*Pneumatically balanced relief valve*

success is attributed to the adoption of three fundamental principles:—

1. The use of a ' seatless ' or piston-type valve head in which the axial pressure forces do not change with valve opening.
2. The substitution of pneumatic loading for spring loading.
3. As a corollary to 2, the adoption of a type of diaphragm whose ' effective ' area remains sensibly constant over a wide range of axial travel.

A sectional drawing of the valve is shown in Fig. 2 and it can be seen that the cylindrical body, which is machined from light alloy, is bored from both ends, having the upper and larger bore counter-bored for approximately half its depth and the lower bore recessed to form a relief chamber. The transverse bore through the chamber is threaded to accommodate the relief pipe unions, and a steel cylinder sleeve containing two outlet ports is inserted vertically through the centre of the chamber. The bottom end cap which also forms the inlet connection to the valve has a central spigot to retain the cylinder sleeve and to locate the end cap in the body, a groove around the body of the spigot housing the lower sealing ring.

A double-ended piston, the head of which is of much larger diameter than the lower part, operates in the cylindrical sleeve and upper bore. The smaller end of the piston is a smooth-finished sliding fit in the cylinder, and the larger head of the piston has a clearance of approximately .1 inch in the upper bore. A pressure-tight seal is made in the upper bore by a specially developed diaphragm which has become known as the ' W ' seal, the beaded edges of which are held in recesses in the piston head and cylinder wall, the flat web of the diaphragm being doubled on itself between the piston and cylinder. The pressure applied above the piston keeps the faces of the diaphragm apart and thus there is very little frictional resistance and a positive seal is maintained. The beaded outer edge of the diaphragm is held against the shoulder in the upper bore by a retaining collar which is held down by the valve head, and the inner beaded edge is secured by a ring screwed to the top of the piston head. A spigot on the underside of the valve head is recessed to house the upper sealing ring and also locates the head on the collar and valve body. A union for connecting the controlling

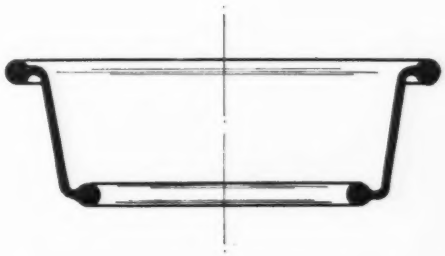


Fig. 3.—Section through diaphragm as moulded

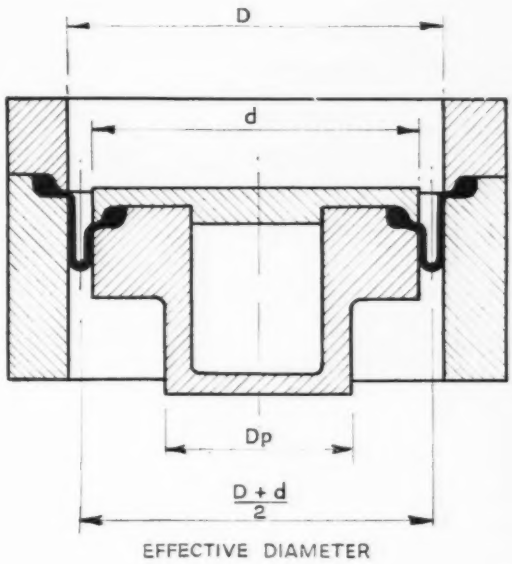


Fig. 4.—Section through diaphragm as installed, illustrating the effective diameter

air supply pipe is fitted into the central boss of the head, adjacent to which a small bleed screw incorporating a needle valve is provided. The bleed screw allows the controlling air to escape at a predetermined rate on a constant loss principle, and is also adjustable for conditions where economy of air supply has to be considered. This leakage allows the piston to rise when it is desired to reduce or cut-off the controlling air pressure in order to reduce the fluid pressure. The internal web of the valve body separating the relief chamber from the

upper bore is provided with two balance holes to prevent hydraulic locking below the piston head.

The special diaphragm used in this valve consists virtually of two sealing rings of different diameters connected by a thin web, the actual length and thickness of the web being dependent on the stroke of the piston and the maximum controlling pressure to be used. The diaphragm is a one-piece moulding and its manufactured form is that of a truncated cone with beaded edges (Fig. 3). The web for this particular valve is .02 inch thick, the material chosen being Neoprene due to the fact that the valve was required for operation with kerosene, a Shore hardness of 60° - 70° being found most suitable. Other plastic and rubber materials may be used depending on the liquid for which the valve is designed to operate, and for controlling pressures above 500 - 600 p.s.i. fabric reinforcing should be moulded integrally with the diaphragm.

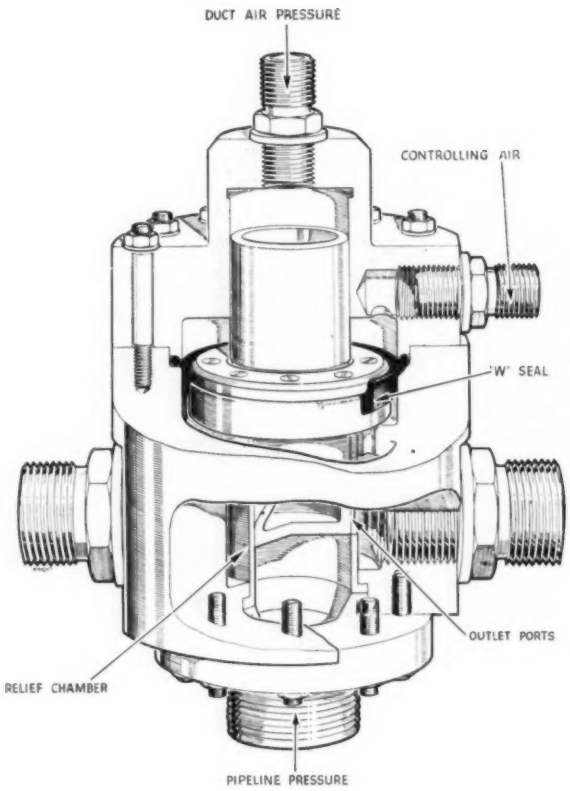


Fig. 6.—Differential relief valve

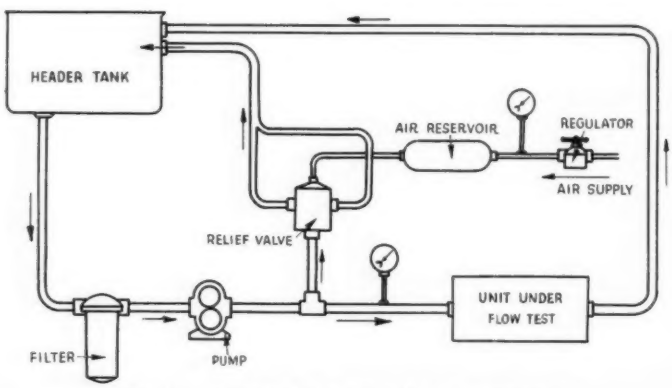


Fig. 5.—A typical test rig with a pneumatically balanced relief valve controlling pump delivery pressure

The operating principle of the valve is very simple. The fluid pipe-line pressure to be controlled is communicated to the face of the small piston, and the controlling air pressure to the top of the large piston and diaphragm. The relative areas are pre-determined by the range of fluid control pressures required and the air pressure available as the controlling medium. The 'effective' diameter of the diaphragm, as defined in Fig. 4, is used in computing the area ratios. In the example illustrated in Fig. 2 the effective diameter of the diaphragm is 2.66 inches and the diameter of the fluid control

piston is 1.13 inches, giving an area ratio of:—

$$\frac{2.66^2}{1.13^2} = 5.55:1$$

This particular valve therefore allows the fluid pressure to be 550 p.s.i. with a maximum controlling air pressure of 100 p.s.i. The fluid supply pumps used with these valves are run at constant speed, and liquid surplus to the requirements of the metering unit, fuel system or other device on test, enters the relief chamber through the ports in the cylinder sleeve and from there passes back to the supply tank. As the ports are partially covered by the lower edge of the smaller piston, the amount of liquid relieved is governed by the amount the piston will lift against the air pressure above the large piston. Thus the piston is always in the 'live' condition and in control of the pipe-line pressure.

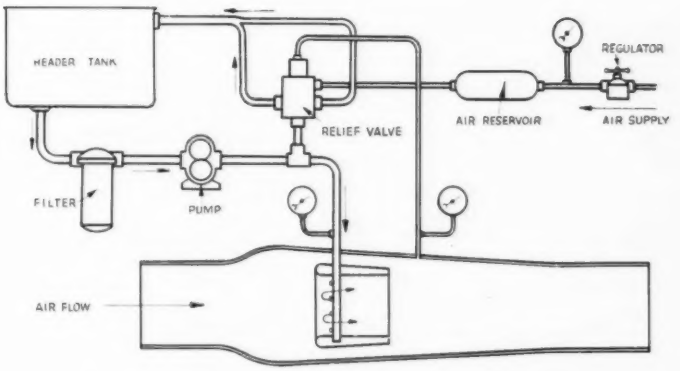


Fig. 7.—A typical combustion rig with a differential relief valve maintaining fuel delivery pressure at a constant differential above duct pressure

As a matter of interest it is possible that valves with reversed ratio pistons could be constructed for low pressure systems, thus permitting the accurate control of system pressures lower than that of a convenient air supply.

As an example of the type of work to which the pneumatically balanced relief valve may be applied, a typical installation in a rig is shown in Fig. 5. A tee piece and a short length of pipe connects the valve to the pump delivery pipe-line, and two relief pipes return the excess liquid to the header tank. For controlling purposes, compressed air is supplied through a pressure regulator and an air reservoir which serves to smooth the supply and prevent the piston 'hunting.' In this case the valve is shown controlling the pump delivery pressure; it has proved that it can handle fluids at flows of up to 5,000 g.p.h. at 150 to 550 p.s.i. and it is thought that higher figures are easily obtainable.

As a further development from the valve already described, the differential relief valve has been manufactured and successfully tested. This valve embodies all the qualities of the original type but is specially designed to maintain a pre-determined differential between two systems, for example, between the air pressure in a duct and a fuel system feeding the duct. The sectional drawing of the valve in Fig. 6 shows that the principal differences are the extension above the large piston of a second smaller piston of the same diameter as the liquid control piston, and the provision of an extra air connection on the side of the body which communicates with the larger cylinder. A typical installation of this valve is show in Fig. 7, the duct air pressure being admitted to the top small cylinder and the fuel supply pipe-line pressure to the lower cylinder. The relative controlling air pressure required to maintain the differential is supplied to the larger cylinder by the side connection. Thus the piston is pneumatically balanced in the same manner as in the original valve. This differential valve has been used with success on experiments in wind tunnels and in conjunction with tests on combustion heaters for aircraft.

Both the pneumatically balanced relief valve and the differential relief valve were designed by the Fluid Control Development Group of D. Napier & Son Limited, Flight Development Establishment, Luton. They were developed for a particular application, so that only a small quantity of each type has been manufactured. However, the success and versatility of these valves have shown that they fulfil an established role in all work connected with fluid control systems and are capable of much further development in both capacity and function.

The Flow of Power and Reactive Components in Rectifier and Inverter Equipments

By T. E. CALVERLEY, B.Sc., A.M.I.E.E., Chief Engineer, Rectifier Department.

PART ONE

The successful application of rectifiers to the conversion of alternating current into direct current has extended over a wide field of electrical engineering. At one end of the scale is electrochemical duty for which mercury arc rectifier installations supply up to 100,000 amperes at voltages as high as 800 volts, while at the other end hard valves and metal rectifiers are used to deliver a few milliamperes. Between these two examples an extensive range of currents and voltages is necessary to meet the requirements of steel mills, factories, high-power radio and television transmitters, nuclear particle accelerating machines, battery charging, traction substations and many other applications.

A few examples of equipments incorporating mercury arc rectifiers are illustrated in Figs. 1 to 4. Traction continues to demand rectifiers operating at from 600 volts to 3,300 volts; Fig. 1 shows a traction equipment rated at 4,030 kW 1,550 volts and comprising four multi-anode sealed mercury arc rectifiers, a cubicle containing smoothing equipment, and a control cubicle. In the higher voltage range, multi-anode sealed steel mercury arc rectifiers (Fig. 2) tend to be superseded by sealed glass excitrons of which Fig. 3 shows a typical example; this unit is rated at 260 kW 15,000 volts for feeding a radio transmitter. Glass excitrons are also used for relatively low power working, as in Fig. 4 which illustrates a single-unit equipment rated at 60 kW 600 volts in which sections of the withdrawable chassis have been cut away for demonstration purposes.

The rectifiers shown in these illustrations are all fitted with control grids which can be used to obtain a variable D.C. output voltage from a fixed A.C. supply voltage. While grid control is a convenient and relatively cheap method of achieving a variable D.C. voltage, it is important that the resulting effects are clearly understood.

Each type of rectifier, whether of the mercury pool cathode type, hot cathode or thyratron, metal rectifier such as selenium or copper oxide, or any other type, has its own characteristic application.

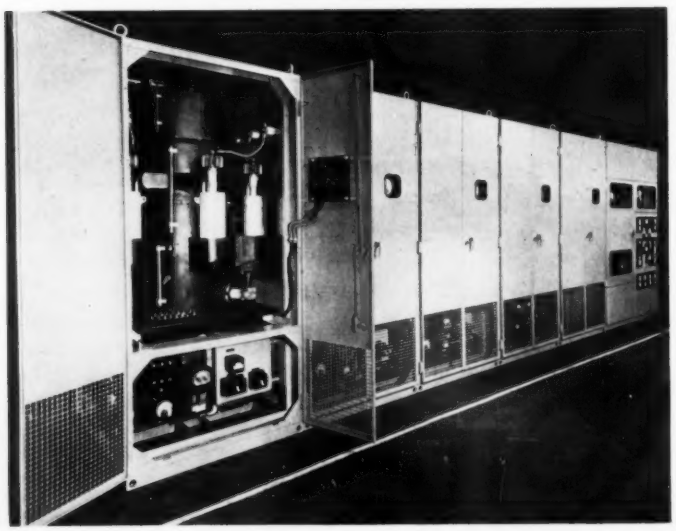


Fig. 1. *A* 4,030 *kW* 1,550 *volts D.C. sealed steel tank rectifier equipment for traction service*

Fig. 2. *Sealed steel tank rectifier with a D.C. output at* 15,000 *volts*

Irrespective of the type however, in general the effect of the rectifier is to allow selected sections of the A.C. voltage to appear in the D.C. circuit in such a way that the addition of all the successive sections contributed by each rectifier anode combine to form the D.C. voltage. The current thereby delivered by the rectifier is continuous, while in each anode circuit it appears as a block of current of definite duration and magnitude depending on the rectifier transformer connections and the value of the direct current. The blocks of anode current are reflected through into the primary circuit and combine to form the line current drawn from the source of A.C. voltage. Power is therefore made to flow from an A.C. source to the D.C. load, but a cursory study of the currents and voltages in the various parts of the circuit does not reveal the true significance of the manner in which the power transfer from the A.C. to the D.C. side is achieved.

It is the object of this article to indicate an approach to the subject by mathematical analysis which may help in appreciating the significance of rectifier operation.

INTRODUCTION TO THEORY

The theory expressed in this article has necessarily been kept simple, but it forms the basis on which elaboration can be made to account for certain effects encountered in practice, such as resonance in the A.C. system, unbalance of rectifier phase groups, and discontinuous direct currents under large angles of grid delay.

It is not a complete general rectifier theory, as it has been compiled to illustrate the application of *secondary or anode current analysis* to circuit calculation, with special reference to the flow of power. The present interest in high-voltage direct-current transmission has prompted the extension of the section on grid control into the field of inversion.

Although in certain cases the mathematical approach has been indicated, no attempt has been

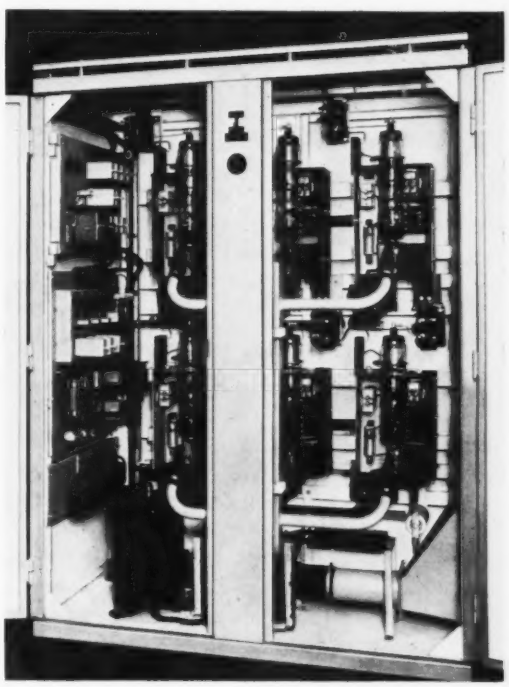


Fig. 3. *A* 260 *kW* 15,000 *volts rectifier unit, comprising six excitrons in 3-phase full wave connection, for feeding a radio transmitter*

Fig. 4. A 60 kW 600 volts single-unit rectifier equipment, comprising three excitron tubes mounted with cooling and auxiliary gear on a removable chassis

made to include the full mathematical calculations but only to quote the resulting equations.

An understanding of the theory is facilitated by considering only one rectifier anode per secondary phase of the transformer, but it will be realised that in practice there may be many rectifier anodes in parallel. To avoid misunderstanding, analysis of the transformer secondary current is carried out and the results can, of course, be divided by the number of anodes in parallel on each secondary phase in order to obtain the constituent components of each anode current.

Method of Approach

To illustrate the method of approach, the simple three-phase inter-star connection shown in Fig. 5 will be used and it will be shown later how the method is extended to the more complicated rectifier circuits.

Consider an instant shortly after the arbitrary zero in Fig. 6, say at T_1. Current is flowing from anode A through the D.C. reactor and load circuit and back to the transformer neutral point N. If it is assumed that the D.C. reactor is of infinite inductance, then this current will be of constant amplitude although the output D.C. voltage is not quite constant. At instant T_2 anode B will become more positive than anode A and current will immediately transfer to anode B if it is further assumed that the rectifier transformer and A.C. supply have zero reactance. Anode B will continue to carry current until T_3 when anode C will take over. Due to this current transfer between anodes, or commutation as it is normally called, the current in the secondary of the transformer, in the primary of the transformer and in the line will be of the wave shape given in Fig. 6.

Consider now the four main parts of the circuit. Firstly, the D.C. load circuit has a pure direct current but a voltage formed by the envelope of the tops of the sine waves in Fig. 6. Secondly, each secondary winding has an A.C. voltage and a rectangular block of current. Thirdly, the primary of the transformer has an A.C. voltage and stepped current wave. Fourthly, the A.C. supply has also an A.C. voltage and stepped current wave. How then is the power transmitted in each of these circuits ?

The method of approach will be to lay down certain basic conditions for determining the current in a secondary winding as follows:—

$$\text{Current amplitude} = I_d = \frac{I_{dc}}{m} \quad \ldots\ldots (1)$$

$$\text{Conduction time} = t = \frac{2\pi}{p} \quad \ldots\ldots (2)$$

where I_d = peak current per secondary winding

I_{dc} = total D.C. load current

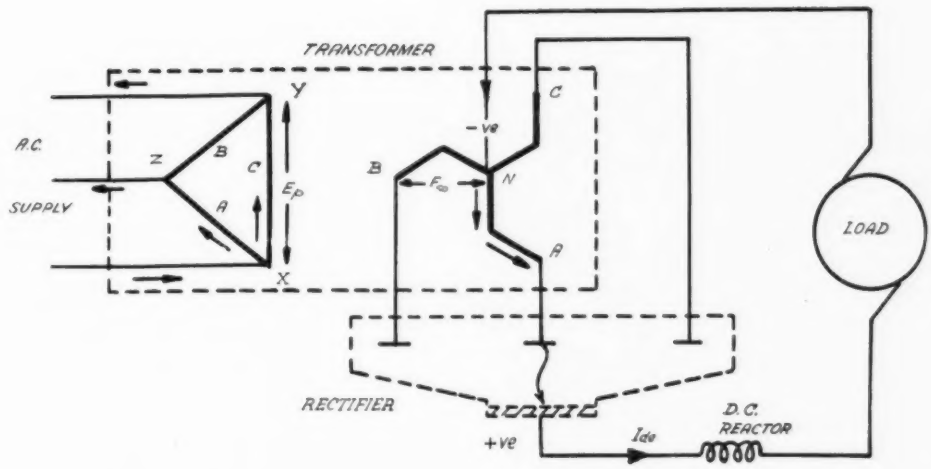


Fig. 5. Connection diagram for a simple three-phase inter-star connected rectifier

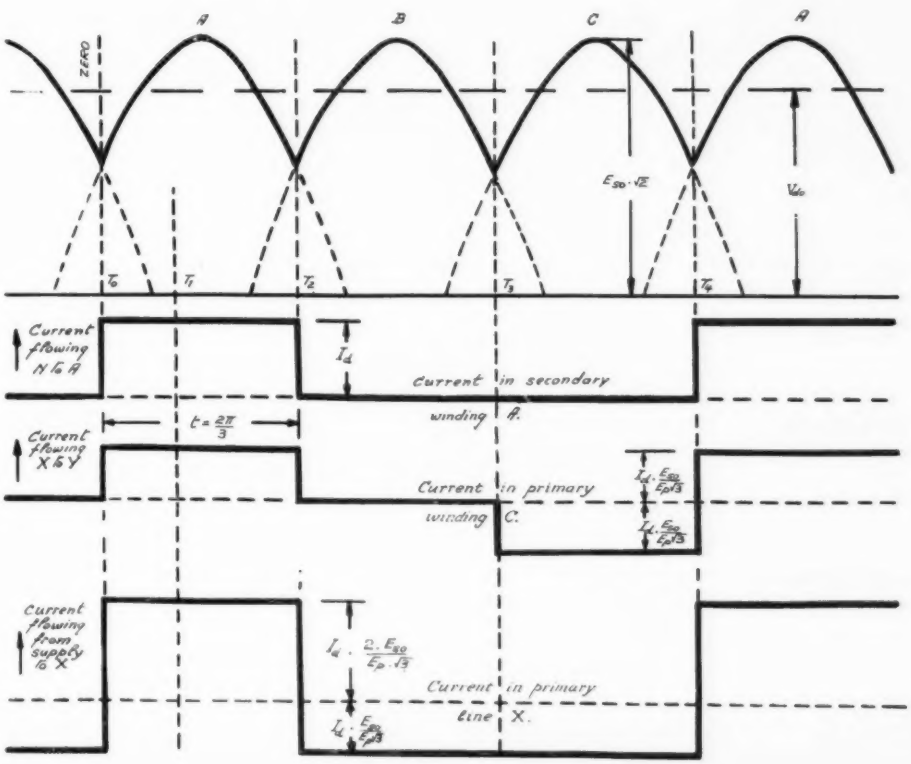


Fig. 6. Current in various parts of the circuit shown in Fig. 5

m = number of rectifier phase groups in parallel
t = conduction time of each anode
p = number of phases per rectifier group

The factor m is introduced since in general there is more than one rectifier group in parallel, as shown in Table IV. However, in certain cases two groups can be considered to act in series as in the so-called Bridge connections (Table III) where it is necessary to treat I_d as equal to I_{dc} in the following analysis.

In the example illustrated in Fig. 5 the factor m is unity and the conduction time is equal to $2\pi/3$. The secondary current can be completely defined by these two factors, and Fourier analysis of the current wave into its components can proceed. Since the components are all continuous functions, an equivalent current circuit may be drawn in which the rectifier is eliminated and all three secondary phases are short-circuited as shown in Fig. 7[1]. Further, since all the components of the secondary current are continuous functions they can be reflected through the transformer to form the primary and line components. Therefore the simple analysis of a single secondary current waveform allows the current components in any part of the circuit to be easily calculated. In addition, since the majority of rectifier circuits are made up of three phase groups, the one simple analysis is a powerful aid in harmonic current calculations in some of the more complicated transformer connections.

Since this article is concerned with power flow, it is desirable to state the two main processes which apply, namely:—

(*a*) D.C. flow of power which is characterised by a direct current associated with a D.C. voltage.

(*b*) A.C. flow of power which is characterised by an alternating current associated with an A.C. voltage of the same frequency and having a component of current in phase with the voltage.

On the basis of secondary current analysis the power flow in the various parts of the circuit will be studied and an endeavour made to show how the theory applies.

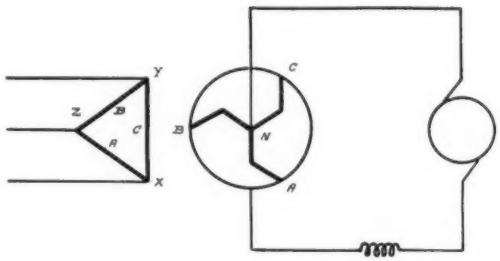


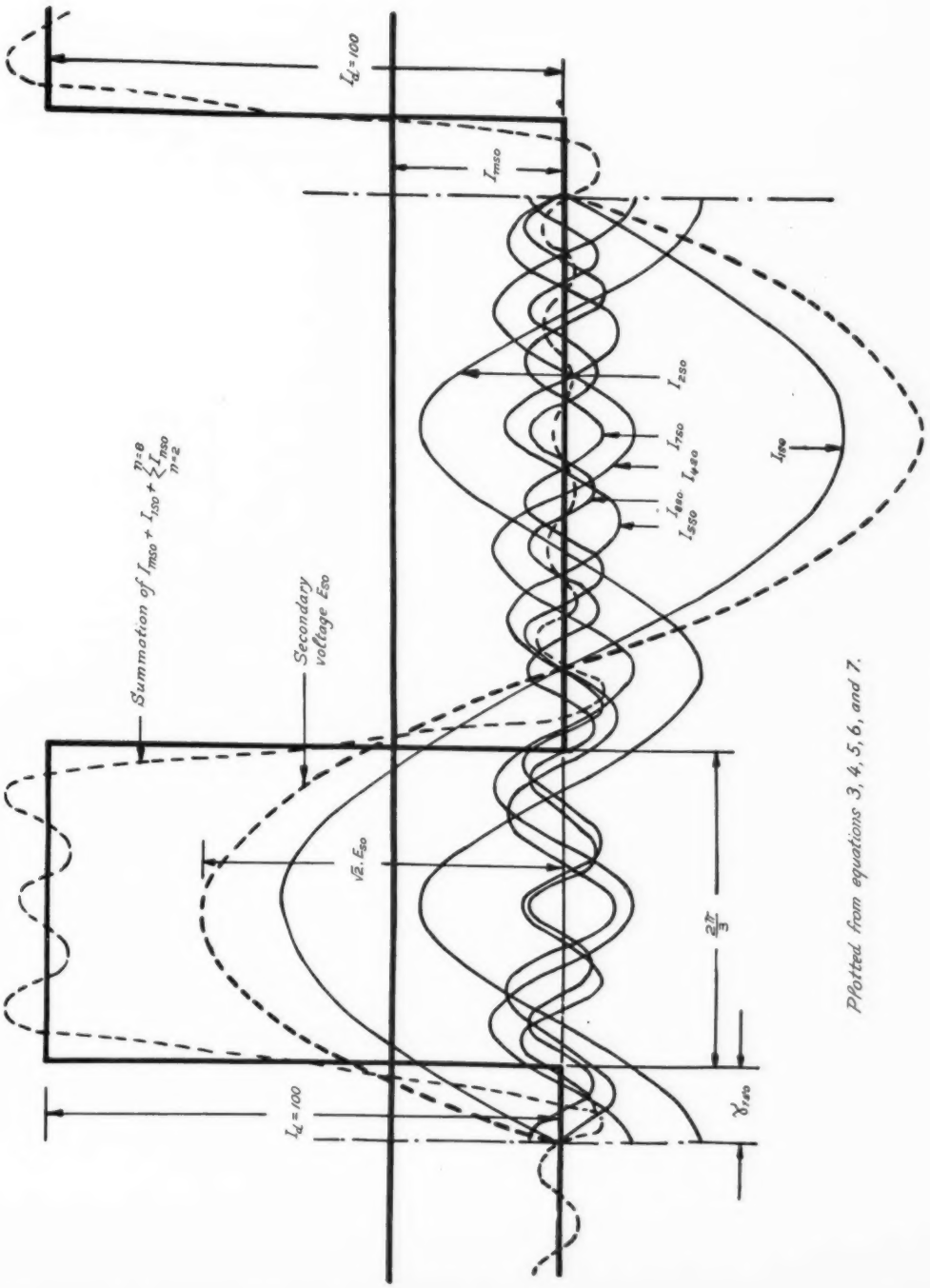
Fig. 7. Equivalent current circuit diagram

Division of Article

The article is divided into three main sections to assist in appreciating the calculations. Ideal conditions will be assumed in the first section to enable the basis of the theory to be laid. In the second section a finite reactance in the transformer and A.C. supply will be introduced and the consequent corrections made to the results of the first section. Finally, in the third section grid control and A.C. reactance will be combined to give additional correction factors. The effects of rectifier arc drop, transformer magnetising current and resistance will be introduced as the theory is developed, and specimen calculations will be made to illustrate the theory. The significance of transformer magnetising current in relation to power factor will be considered at the end of the article.

Throughout the calculations the assumption of infinite inductance on the D.C. side will be maintained, since in many practical applications the inductance of the load combined with the equivalent inductance of the rectifier transformer is sufficiently high to render the practical conditions very similar to those calculated on the basis of infinite inductance The factor which determines whether the practical conditions depart from the infinite inductance assumption is whether the vector sum of the harmonic currents set up in the D.C. load as a result of harmonics in the rectifier output voltage are an appreciable percentage of the direct current. Obviously this is a function of the actual harmonic voltages, the impedance of the D.C. circuit at each harmonic frequency and the direct current in question. Grid control causes a large increase in the harmonics in the D.C. voltage which may under

[1] *The short-circuited transformer secondary is of course a mathematical manipulation only and not a practical condition. In the more complicated connections to follow, the equivalent current circuit will be derived by short-circuiting together all secondary windings. Where there is more than one secondary star point, as in inter-phase transformer or fork connections, it is also necessary to consider the star points joined together.*



Plotted from equations 3, 4, 5, 6, and 7.

Fig. 8. Harmonic analysis of an anode current

certain conditions produce harmonic currents having a net peak value in excess of the direct current, thereby leading to the condition of discontinuous current on the D.C. side. However, in the section on grid control the assumption of infinite inductance will be maintained and a consideration of the validity of this assumption will be made in the Appendix at the end of the article.

Study of rectifier circuits has to be made by assuming simple conditions to enable the theory to be calculated and then, using a step by step process, each assumption has to be considered and the simple theory modified accordingly. As the corrections are introduced, the initial conditions have to be examined to see if any of the original assumptions on which the theory is based have been thereby rendered invalid.

To facilitate the description, the side of the transformer connected to the rectifier is called the secondary while the side connected to the A.C. supply system is called the primary.

Notation

The following symbols are used throughout this article:—

Direct current output	I_{dc}
Direct current per phase group	I_d
Number of phase groups	m
Number of transformer phases per group	p
Conducting time of each anode	t
Harmonic number	n
Primary voltage	E_p
Any integer	k
Conversion constant	K
Three sections of secondary currents	i_1, i_2, i_3
Peak secondary short-circuit (commutating) current	I_s
Power loss due to circuit copper	P_{cu}
Power loss due to arc loss	P_{arc}
Increment of D.C. voltage lost due to $P_{cu}+P_{arc}$	ΔV_{dc}
Percentage reactance of total A.C. circuit referred to primary kVA (commutation basis) including the rectifier transformer	$\%X_c$
Angle of grid delay	α
Transformer magnetising current	I_e
Equivalent inductance of rectifier transformer	L_T

The following symbols are used for the assumed conditions in Sections I, II and III:—

Quantity	Sectn. I	Sectn. II	Sectn. III
R.M.S. secondary voltage	E_{so}	$E_{s\mu}$	$E_{s\alpha}$
D.C. output voltage	V_{do}	$V_{d\mu}$	$V_{d\alpha}$
D.C. component of secondary current	I_{mso}	$I_{ms\mu}$	$I_{ms\alpha}$
R.M.S. fundamental component of secondary current	I_{1so}	$I_{1s\mu}$	$I_{1s\alpha}$
R.M.S. harmonic component of secondary current	I_{nso}	$I_{ns\mu}$	$I_{ns\alpha}$
Phase angle of fundamental component of secondary current	γ_{1so}	$\gamma_{1s\mu}$	$\gamma_{1s\alpha}$
Phase angle of harmonic component of secondary current	γ_{nso}	$\gamma_{ns\mu}$	$\gamma_{ns\alpha}$
R.M.S. fundamental component of primary current	I_{1po}	$I_{1p\mu}$	$I_{1p\alpha}$
R.M.S. fundamental component of line current	I_{1lo}	$I_{1l\mu}$	$I_{1l\alpha}$
R.M.S. harmonic component of primary current	I_{npo}	$I_{np\mu}$	$I_{np\alpha}$
R.M.S. harmonic component of line current	I_{nlo}	$I_{nl\mu}$	$I_{nl\alpha}$
D.C. power output	P_{do}	$P_{d\mu}$	$P_{d\alpha}$
Angle of overlap	—	μ_o	μ_α
R.M.S. fundamental component of secondary current (power component)	—	$I_{1s\mu_p}$	$I_{1s\alpha_p}$
R.M.S. fundamental component of secondary current (reactive component)	—	$I_{1s\mu_r}$	$I_{1s\alpha_r}$
Reactive power	—	$P_{1s\mu_r}$	$P_{1s\alpha_r}$
Fundamental current component correction factor	—	$\omega(1\mu)$	$\omega(1\alpha)$
Power component of fundamental correction factor	—	$\omega(1\mu)p$	$\omega(1\alpha)p$
Reactive component of fundamental correction factor	—	$\omega(1\mu)r$	$\omega(1\alpha)r$
Phase angle of fundamental component of primary current (corrected for transformer magnetising current)	—	$\gamma_{1p\mu_c}$	$\gamma_{1p\alpha_c}$

R.M.S. value of nth voltage harmonic .. .. V_{ndo} $V_{nd\mu}$ $V_{nd\alpha}$

R.M.S. nth harmonic voltage correction factor .. .. — $\omega(n\mu)d$ $\omega(n\alpha)d$

I.—SIMPLE RECTIFIER THEORY

Assumptions

The following assumptions are made in this section:—

(*a*) Zero reactance of transformer and A.C. supply.

(*b*) Zero grid delay.

(*c*) Zero rectifier arc drop.

(*d*) Infinite inductance of D.C. load.

(*e*) Zero transformer and circuit resistance.

(*f*) Zero transformer magnetising current.

Analysis of Secondary Current Wave-form

Consider the general case of a secondary rectangular current wave-form of amplitude I_d and duration $2\pi/p$ as laid down by equations 1 and 2 in the Introduction (Method of Approach). Application of Fourier analysis to this wave-shape gives the following components:—

(*a*) D.C. component

$$I_{mso} = \frac{I_d}{p} \quad \ldots\ldots (3)$$

(*b*) A.C. component of fundamental frequency of R.M.S. value

$$I_{1so} = -\frac{I_d\sqrt{2}}{\pi} \sin\frac{\pi}{p} \quad \ldots\ldots (4)$$

(*c*) Certain harmonic components of R.M.S. value

$$I_{nso} = \frac{I_d\sqrt{2}}{n\pi} \sin\frac{n\pi}{p} \quad \ldots\ldots (5)$$

The value of I_{1so} is obtained by equating 'n' to unity in equation 5.

Although the above equations give the magnitude of the components, it is necessary to derive their phase in order to give a complete specification. A convention has been adopted in which an A.C. component is said to be positive when it is flowing in the secondary winding towards the rectifier anode, and its phase angle is expressed as the angle at which this current goes positive in advance of the zero, instant T_o in Fig. 6. The Fourier analysis shows that:

for the fundamental,

$$\text{Tan } \gamma_{1so} = \cot\frac{\pi}{p} \quad \ldots\ldots (6)$$

and for the harmonics,

$$\text{Tan } \gamma_{nso} = \cot\frac{n\pi}{p} \quad \ldots\ldots (7)$$

The five equations 3, 4, 5, 6 and 7 allow all the components of the secondary current to be calculated in magnitude and located in phase. To check the result of such calculations a block of current of 100 amperes peak and a duration of 120° (corresponding to three-phase) was analysed using the five equations, and the components were then plotted in correct phase and with correct amplitudes to give, when added, the dotted curve in Fig. 8. Since the harmonics were considered only up to the 8th, the correspondence between the dotted curve and block of current showed the analysis to be correct.

Equation 5 is of interest since it shows that for any harmonic of order n equal to p, the amplitude is zero, *e.g.* in the case taken of p equal to 3, all harmonics except multiples of 3 are present. This point will be of greater significance when harmonic loading of the A.C. supply is considered.

Significance of Secondary Current Analysis

Returning to the equivalent current circuit shown in Fig. 7, since the discontinuous block of secondary current can be replaced by a whole series of continuous functions as in Fig. 8 (once I_d and t have been specified) it can be assumed that the rectifier and the block of secondary current are replaced by all the continuous currents in all secondary phases, displaced by time t. This leads to the following further results.

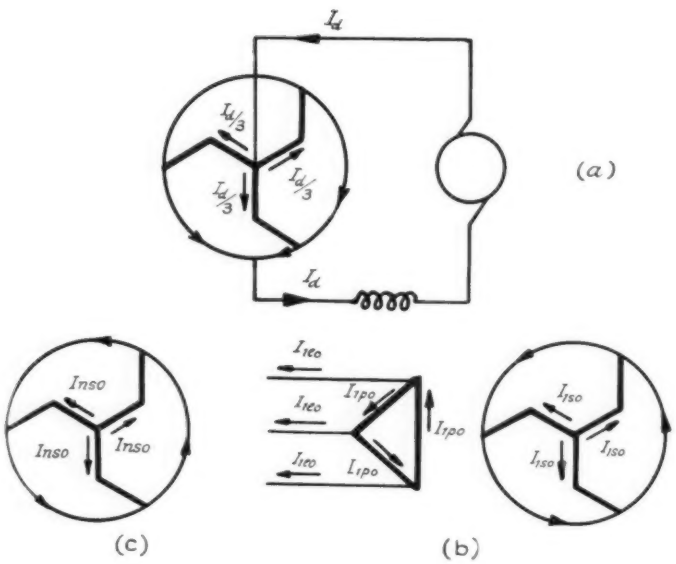


Fig. 9. Equivalent circuit paths for (a) D.C. component (b) A.C. component of fundamental frequency (c) certain harmonic components

D.C. Component

From equation 3 the D.C. component of each secondary current is $I_d/3$ and these components unite to form the total direct current. Since each leg of the transformer considered in Fig. 5 has two windings, the D.C. component of current will produce no net ampere-turns on the transformer core and will consequently have no saturating effect. If a simple star secondary is used, the D.C. component ampere-turns would act in the same direction on each leg and produce a flux with a return path through the transformer tank. To eliminate this flux from the core the inter-star is preferable. In each secondary winding the only voltage is the fundamental A.C. voltage which cannot represent any power flow when taken in conjunction with the D.C. component of current. Therefore, the D.C. component of current serves no useful purpose in the power flow in the secondary circuit and is purely a circulating component which adds to the transformer kVA. Since this component is D.C. it cannot appear in the primary side. The path followed by the D.C. component is shown in Fig. 9*a*.

A.C. Component of Fundamental Frequency

The component of fundamental frequency I_{1so} as given by equation 4 is displaced by 120° in each of the three secondary windings so that the vector sum is zero, *i.e.* I_{1so} circulates in the secondary circuit and does not appear in the load. This is consistent with the presence of the infinitely inductive D.C. reactor. However, the important point is that since this component forms a balanced 3-phase system, the currents may be reflected into the primary and line as in a normal 3-phase A.C. system, permitting the calculation of the fundamental frequency component of current in the primary as:—

$$I_{1po} = I_{1so}.\frac{E_{so}}{E_p} = \frac{I_d\sqrt{2}}{\pi}\sin\frac{\pi}{p}.\frac{E_{so}}{E_p} \quad . . (8)$$

and the corresponding line component as:—

$$I_{1lo} = \sqrt{3}.\frac{I_d\sqrt{2}}{\pi}\sin\frac{\pi}{p}.\frac{E_{so}}{E_p} \quad . . (9)$$

Reference to Fig. 9*b* will show the general circuit followed by the fundamental component of current. This component is of major importance, as in secondary, primary and line there is a voltage of the same frequency, and it will be apparent that this is the power-carrying component. This aspect is studied in more detail in the subsequent subsection headed 'Practical Example' on page 221.

Harmonic Components

The harmonic components have been shown to be present in the secondary current having every order except any multiple of p, *i.e.* for the three-phase example harmonics 2, 4, 5, 7, 8, 10, 11, etc., will be present. For each of these harmonics the factor $\sin\frac{n\pi}{p}$ in equation 5 is not constant for all values of n, so that in the secondary winding the

amplitude of each harmonic is given by:—

$$\text{R.M.S. value of } n\text{th harmonic} = \frac{\text{R.M.S. value of fundamental}}{n} \times \frac{\sin\frac{n\pi}{p}}{\sin\frac{\pi}{p}} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

Thus, the amplitude of the harmonic in general decreases as the order increases. The path followed by these harmonics will now be traced as for the previous components. Consider the 4th harmonic and the two sections of secondary winding on one transformer limb. Each winding has a 4th harmonic given by equation 5 and spaced 120 fundamental electrical degrees but reversed in direction relative to the primary windings. A little calculation will show that the effect of these two currents is to produce a primary component of $I_{4so} \cdot \frac{E_{so}}{E_p}$. Consideration of the other harmonics will show that they can be similarly treated to give the relation:—

R.M.S. value of nth current harmonic in *primary* side

$$I_{npo} = I_{nso} \cdot \frac{E_{so}}{E_p} \quad \ldots\ldots\ldots (11)$$

and R.M.S. value of nth current harmonic in the *line*

$$I_{nlo} = \sqrt{3} \cdot I_{nso} \cdot \frac{E_{so}}{E_p} \quad \ldots\ldots\ldots (12)$$

In the specific case chosen, one property of harmonic flow was not brought out as it does not apply to the simple inter-star connection. To illustrate this point a further example with the well-known double star connection shown in Fig. 10 will be used. The equivalent diagram for this connection is shown in Fig. 11 and is in effect just a complete short-circuit applied to the transformer with the neutral points joined. Since the double star connection acts like two three-phase groups in

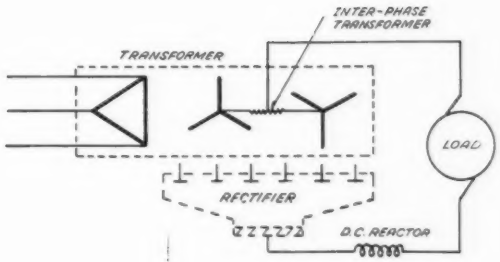


Fig. 10. Double-star connection

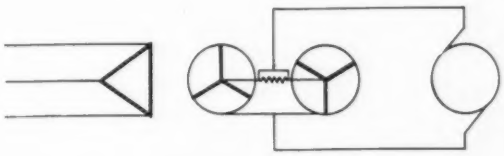


Fig. 11. Equivalent double-star connection

parallel, the value of p is 3 and the value of m is 2. Hence the peak current per group I_d is equal to $I_{dc}/2$. The basic conditions laid down in the Introduction are therefore as already worked out for the inter-star case, only the peak current is half the direct-current.

This leads to the result that the D.C. component, fundamental component and harmonic components of secondary current are as before, and the same treatment can be applied to give the components in any part of the circuit. However, in considering the harmonics a new condition arises in that since the two secondary windings on the same limb are 180 fundamental degrees apart, all even harmonics will produce equal and opposite ampere-turns relative to the primary winding and will therefore produce no net primary current of their order. Two conditions of harmonic flow can therefore be postulated:—

(*a*) *Harmonic transmission* in which the harmonic ampere-turns in the two secondary windings will add to produce *net* primary harmonic currents.

(*b*) *Harmonic cancellation* in which the harmonic ampere-turns in the two secondary windings will act to produce *no* primary harmonic current.

TABLE I

CURRENT BALANCE FOR DOUBLE-STAR CONNECTION

SECONDARY AND D.C. COMPONENT AS PERCENTAGE OF I_d. PRIMARY AND LINE AS PERCENTAGE OF $I_d \frac{E_{so}}{E_s}$.

Circuit	Harmonic Number													I_{rms}	m	p	$m.p$
	1	2	3	4	5	6	7	8	9	10	11	12	13				
Secondary	39	19.5	—	9.7	7.8	—	5.6	4.9	—	3.9	3.5	—	3.0	33			
Primary	78	—	—	—	15.6	—	11.1	—	—	—	7.1	—	6.2	—	2	3	6
Line	135	—	—	—	27	—	19.3	—	—	—	12.3	—	10.3	—			

By this process a current balance has been compiled in Table I for the double star connection. It will be apparent from later sections in this article that the process of 'cancellation' is the means whereby several groups of three-phase units may operate in parallel and maintain the same secondary current wave-form yet cause the elimination of certain primary harmonics.

The procedure so far developed in this section has been applied to the commonly-used connections resulting in Tables II, III and IV which relate to the three main categories of rectifier connections, namely:—

(*a*) Table II Straight or half-wave operation.

(*b*) Table III Bridge connection or full-wave.

(*c*) Table IV Inter-phase transformer connection.

The significance of the secondary current analysis will now be apparent in that the simple results obtained in the sub-section headed 'Analysis of Secondary Current Wave Form' on page 213 allow calculation of the component currents in any part of any rectifier circuit.

Application of equations 10, 11 and 12 and allowing for harmonic 'cancellation' leads to the well-known harmonic laws which are normally derived by other means[2]:—

(*a*) *Law of harmonic numbers.* In the primary side of an '*m.p*'-phase rectifier system the current harmonics present will be of the order

$$n = k.m.p. \pm 1 \quad \ldots\ldots\ldots (13)$$

where k is any integer.

(*b*) *Law of harmonic amplitudes.* The R.M.S. value of the primary current harmonics in an '*m.p*'-phase rectifier system is given by

$$I_{npo} = \frac{I_{1po}}{n} \quad \ldots\ldots\ldots\ldots (14)$$

Derivation of Transformer kVA

In normal power-transformer practice the kVA on either side of the transformer is equal, but this is not so in rectifier transformers as is easily shown by a simple calculation of the R.M.S. currents using the stepped current wave shapes. However, the principles of this article may be used to illustrate the true significance of the differences in kVA. The R.M.S. value of a complex current wave may be derived from:—

$$I_{rms} = \sqrt{(I_m^2 + I_1^2 + I_2^2 \ldots I_n^2)} \quad \ldots (15)$$

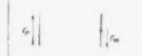
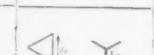
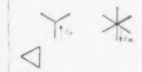
where I_{rms} = R.M.S. current.
I_m = D.C. component, if any.
I_1 = R.M.S. fundamental component.
$I_2 \ldots I_n$ = R.M.S. harmonic components.

In the secondary winding of a rectifier transformer the D.C. component and certain harmonics flow which do not appear in the primary winding, as already shown, and they consequently add to the secondary kVA. Therefore, the difference in kVA is due to harmonics and the D.C. component which are necessary to make up the required wave-shape but serve no other useful function. It can be shown that in any specific case the R.M.S. current in any part of the circuit, calculated by means of equation 15 and including all the components calculated from equations 3, 4 and 5, is identical with that derived

[2] *H. Rissik. The fundamental theory of arc convertors (Book).*

TABLE II

STRAIGHT CONNECTIONS

Connection		R.M.S. Value of Harmonic Currents Secondary current as a percentage of I_{1L} Primary and line currents as a percentage of I_{1L}. $\frac{E_n}{E_1}$																			
		n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
A		Secondary	45	—	15	—	9·0	—	6·4	—	5·0	—	4·1	—	3·5	—	3·0	—	2·6	—	2·4	
		Primary	90	—	30	—	18	—	12·8	—	1·0	—	8·2	—	7·0	—	6·0	—	5·2	—	4·8	
B		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Primary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Line	67·5	33·7	—	16·9	13·5	—	9·7	8·5	—	6·7	6·1	—	5·2	4·8	—	4·2	4·0	—	3·5	
C		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Primary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Line	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
D		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Primary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Line	67·5	33·7	—	16·9	13·5	—	9·7	8·5	—	6·7	6·1	—	5·2	4·8	—	4·2	4·0	—	3·5	
E		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Primary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Line	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
F		Secondary	22·5	19·5	15	9·7	4·5	—	3·2	4·9	5·0	3·9	2·0	—	1·7	2·8	3·0	2·4	1·3	—	1·2	
		Primary	45	—	30	—	9·0	—	6·4	—	10	—	4·0	—	3·4	—	6·0	—	2·6	—	2·4	
		Line	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·2	—	—	—	4·5	—	4·1	
G		Secondary	39	19·5	x_3	9·7	7·8	x_6	5·6	4·9	x_9	3·9	3·5	x_{12}	3·0	2·8	x_{15}	2·4	2·3	x_{18}	2·0	
		Primary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·2	—	—	—	4·5	—	4·1	
		Line	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·2	—	—	—	4·5	—	4·1	
H		Secondary	22·5	19·5	15	9·7	4·5	—	3·2	4·9	5·0	3·9	2·0	—	1·7	2·8	3·0	2·4	1·3	—	1·2	
		Primary	45	—	—	—	9·0	—	6·4	—	—	—	4·0	—	3·4	—	—	—	2·6	—	2·4	
		Line	45	—	—	—	9·0	—	6·4	—	—	—	4·0	—	3·4	—	—	—	2·6	—	2·4	
		Tertiary	—	—	30	—	—	—	—	—	10	—	—	—	—	—	6·0	—	—	—	—	
J		Secondary	22·5	19·5	15	9·7	4·5	—	3·2	4·9	5·0	3·9	2·0	—	1·7	2·8	3·0	2·4	1·3	—	1·2	
		Inner	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Primary	45	—	—	—	9·0	—	6·4	—	—	—	4·0	—	3·4	—	—	—	2·6	—	2·4	
		Line	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·2	—	—	—	4·5	—	4·1	
K		Secondary	22·5	19·5	15	9·7	4·5	—	3·2	4·9	5·0	3·9	2·0	—	1·7	2·8	3·0	2·4	1·3	—	1·4	
		Inner	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	
		Primary	45	—	—	—	9·0	—	6·4	—	—	—	4·0	—	3·4	—	—	—	2·6	—	2·4	
		Line	45	—	—	—	9·0	—	6·4	—	—	—	4·0	—	3·4	—	—	—	2·6	—	2·4	

* *An interesting property of the diametric connection with STAR primary (connection G) is illustrated by the process of secondary current analysis. Since the connection has six secondary phases and no interphase transformer it would be expected that 'p' would be 6 and 'm' equal to 1, as shown in italics in the appropriate column of this table. This would make the secondary currents each contain harmonics of multiples of 3 as well as other harmonics. These, however, could not flow in the star primary so the mode of operation is automatically changed to eliminate them from the primary by the secondary currents only containing enough of these harmonics to convert the operation to double star (as connection R in Table IV), utilising the leakage reac becomes 3, and 'm' becomes 2. In Ta transformer reactance.*

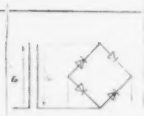
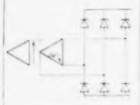
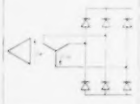
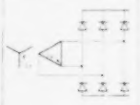
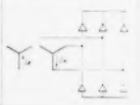
In connection F the harmonics of in the tertiary.

CTIONS

RRENTS rrents as a percentage of I_d. $\frac{E_{SO}}{E_p}$.											D.C. component I_{dc} as percentage I_d	p	m	$p.m$	K
15	16	17	18	19	20	21	22	23	24	25					
3·0	—	2·6	—	2·4	—	2·1	—	2·0	—	1·8	50	2	1	2	2
6·0	—	5·2	—	4·8	—	4·2	—	4·0	—	3·6	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	—	3	1	3	1
—	4·2	4·0	—	3·5	3·4	—	3·1	2·9	—	2·7	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	—	3	1	3	1
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	—	3	1	3	1
—	4·2	4·0	—	3·5	3·4	—	3·1	2·9	—	2·7	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	—	3	1	3	1
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	—				
3·0	2·4	1·3	—	1·2	1·9	2·2	1·8	1·0	—	0·9	16·6				
6·0	—	2·6	—	2·4	—	4·4	—	2·0	—	1·8	—	6	1	6	2
—	—	4·5	—	4·1	—	—	—	3·5	—	3·1	—				
x_{15}	2·4	2·3	x_{18}	2·0	1·9	x_{21}	1·8	1·7	x_{24}	1·6	16·6				
—	—	4·5	—	4·1	—	—	—	3·5	—	3·1	—	3	2	6	2
—	—	4·5	—	4·1	—	—	—	3·5	—	3·1	—	6	1		
3·0	2·4	1·3	—	1·2	1·9	2·2	1·8	1·0	—	0·9	16·6				
—	—	2·6	—	2·4	—	—	—	2·0	—	1·8	—				
—	—	2·6	—	2·4	—	—	—	2·0	—	1·8	—	6	1	6	2
6·0	—	—	—	—	—	4·4	—	—	—	—	—				
3·0	2·4	1·3	—	1·2	1·9	2·2	1·8	1·0	—	0·9	16·6				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	—	2·6	—	2·4	—	—	—	2·0	—	1·8	—	6	1	6	2
—	—	4·5	—	4·1	—	—	—	3·5	—	3·1	—				
3·0	2·4	1·3	—	1·4	1·9	2·2	1·8	1·0	—	0·9	16·6				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	—	2·6	—	2·4	—	—	—	2·0	—	1·8	—	6	1	6	2
—	—	2·6	—	2·4	—	—	—	2·0	—	1·8	—				

R in Table IV), utilising the leakage reactance of the transformer as an interphase transformer. The effective 'p' thus becomes 3, and 'm' becomes 2. In Table II these harmonics are indicated by 'x' since their value depends on the transformer reactance.

In connection F the harmonics of multiples of 3 circulate in the delta primary, and in connection H they circulate in the tertiary.

Connection		R.M.S. Value of Secondary currents as a percentage of I_n Prima											
	n	1	2	3	4	5	6	7	8	9	10	11	
M		Anode	45	—	15	—	9·0	—	6·4	—	5·0	—	4·1	
		Secondary	90	—	30	—	18	—	12·8	—	10	—	8·2	
		Primary	90	—	30	—	18	—	12·8	—	10	—	8·2	
		Line	90	—	30	—	18	—	12·8	—	10	—	8·2	
N		Anode	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	
		Secondary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Primary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Line	135	—	—	—	27·0	—	19·3	—	—	—	12·3	
O		Anode	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	
		Secondary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Primary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Line	135	—	—	—	27·0	—	19·3	—	—	—	12·3	
P		Anode	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	
		Secondary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Primary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Line	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
Q		Anode	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	
		Secondary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Primary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	
		Line	78	-	—	—	15·6	—	11·1	—	—	—	7·1	

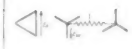
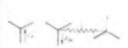
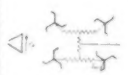
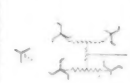
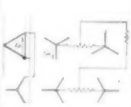
† *This is a special case in which 'p' is 2 and 'm' is 2, but the effective 'm.p' i*

TABLE III
BRIDGE CONNECTIONS

R.M.S. Value of Harmonic Currents nts as a percentage of I_L. Primary and line currents as a percentage of $I_L \cdot \frac{E_{sec}}{E_p}$.																			D.C. component $I_{(av)}$ as percentage I_L	p	m	$p.m$	K
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25					
6·4	—	5·0	—	4·1	—	3·5	—	3·0	—	2·6	—	2·4	—	2·1	—	2·0	—	18	50				
12·8	—	10	—	8·2	—	7·0	—	6·0	—	6·2	—	4·8	—	4·2	—	4·0	—	3·6	—				
12·8	—	10	—	8·2	—	7·0	—	6·0	—	5·2	—	4·8	—	4·2	—	4·0	—	3·6	—	2	2	†2	2
12·8	—	10	—	8·2	—	7·0	—	6·0	—	5·2	—	4·8	—	4·2	—	4·0	—	3·6	—				
5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—	3	2	6	2
19·3	—	—	—	12·3	—	10·3	—	—	—	7·9	—	7·1	—	—	—	5·2	—	5·4	—				
5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—	3	2	6	2
19·3	—	—	—	12·3	—	10·8	—	—	—	7·9	—	7·1	—	—	—	5·2	—	5·4	—				
5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—	3	2	6	2
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				
5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—	3	2	6	2
11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				

† is 2 and 'm' is 2, but the effective 'm.p' is still 2 since the two series groups have no relative phase shift as in all the other connections.

TABLE IV
INTER-PHASE TRANSFORMER CONNECTIONS

Connection		R.M.S. Value of Harmonic Currents Secondary currents as a percentage of I_{L_2} Primary and line currents as a percentage of $I_{L_2} \cdot \frac{E_2}{E_1}$																		
		n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
R		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0
		Primary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1
		Line	135	—	—	—	27·0	—	19·3	—	—	—	12·3	—	10·3	—	—	—	7·9	—	7·1
S		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0
		Primary	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1
		Line	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1
T		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0
		Primary	156	—	—	—	—	—	—	—	—	—	14·1	—	12·0	—	—	—	—	—	—
		Line	270	—	—	—	—	—	—	—	—	—	24·5	—	20·8	—	—	—	—	—	—
U		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0
		Primary	156	—	—	—	—	—	—	—	—	—	14·1	—	12·0	—	—	—	—	—	—
		Line	156	—	—	—	—	—	—	—	—	—	14·1	—	12·0	—	—	—	—	—	—
V		Secondary	39	19·5	—	9·7	7·8	—	5·6	4·9	—	3·9	3·5	—	3·0	2·8	—	2·4	2·3	—	2·0
		Primary ◁	78	—	—	—	15·6	—	11·1	—	—	—	7·1	—	6·0	—	—	—	4·6	—	4·1
		Primary ⋋	135	—	—	—	27·0	—	19·3	—	—	—	12·3	—	10·3	—	—	—	7·9	—	7·1
		Line	270	—	—	—	—	—	—	—	—	—	24·5	—	20·8	—	—	—	—	—	—

CONNECTIONS

ENTS nts as a percentage of $I_{d.c.} \cdot \frac{E_{so}}{E_p}$.											D.C. component $I_{m=o}$ as percentage $I_{d.c.}$	p	m	$p.m$	K
15	16	17	18	19	20	21	22	23	24	25					
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—	3	2	6	2
—	—	7·9	—	7·1	—	—	—	5·9	—	5·4	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—	3	2	6	2
—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	—	—	—	—	—	—	—	6·8	—	6·2	—	3	4	12	4
—	—	—	—	—	—	—	—	11·7	—	10·8	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	—	—	—	—	—	—	—	6·8	—	6·2	—	3	4	12	4
—	—	—	—	—	—	—	—	6·8	—	6·2	—				
—	2·4	2·3	—	2·0	1·9	—	1·8	1·7	—	1·6	33				
—	—	4·6	—	4·1	—	—	—	3·4	—	3·1	—				
—	—	7·9	—	7·1	—	—	—	5·9	—	5·4		3	4	12	4
—	—	—	—	—	—	—	—	11·7	—	10·8	—				

from a simple calculation based on rectangular current wave-forms.

Flow of Power

Returning now to the problem of power flow, it has been explained that the fundamental component of secondary current can be calculated by equation 4, and by the process of 'transmission' it may be reflected through the transformer to the supply network. Exactly how this is done will depend on the particular transformer connection; for instance, assuming a unity ratio of E_{so}/E_p, the value of the fundamental secondary current has to be multiplied by a certain constant. Some examples are tabulated below.

Connection	Diagram	Constant
Single-phase	A in Table II	K = 2
Three-phase star, delta primary	B in Table II	K = 1
Diametric, delta primary	F in Table II	K = 2
Double star, delta primary	R in Table IV	K = 2
Quadruple zig-zag, delta primary	T in Table IV	K = 4

For the various connections the conversion factor K may be derived as being the number of secondary phases which contribute to each primary phase. The value of these factors is given in Tables II, III and IV for each connection. The general equations in the calculation of the R.M.S. value of the fundamental components in each part of the circuit may now be restated as:—

Secondary circuit,

$$I_{lso} = \frac{I_d\sqrt{2}}{\pi}.\sin\frac{\pi}{p} \quad \ldots\ldots\ldots (4)$$

Primary current,

$$I_{lpo} = \frac{I_d\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.K.\frac{E_{so}}{E_p} \quad \ldots (16)$$

Line current,

$$I_{llo} = I_{lpo} \text{ or } \sqrt{3}.I_{lpo} \quad \ldots\ldots (17)$$

depending on whether the primary is star or delta connected.

Before applying these equations to a determination of the power flow it is necessary to note a further well-known expression which relates the D.C. voltage V_{do} to the R.M.S. secondary voltage E_{so}, namely:—

$$V_{do} = E_{so}.\sqrt{2}.\frac{p}{\pi}.\sin\frac{\pi}{p} \quad \ldots\ldots (18)$$

TABLE V

FLOW OF POWER THROUGH RECTIFIER EQUIPMENT

Circuit	D.C. Side	Secondary	Primary	Line (Δ)	Line ($\curlywedge$)
Current component related to power flow ..	D.C.	Fundamental	Fundamental	Fundamental	Fundamental
Value of above component ..	$m.I_d$	$\frac{I_d\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.$	$\frac{I_d\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.\frac{E_{so}}{E_p}.K$	$\frac{I_d\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.\frac{E_{so}}{E_p}.K\sqrt{3}$	$\frac{I_d\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.\frac{E_{so}}{E_p}.K$
Voltage related to power flow	V_{do}	E_{so}	E_p	$\frac{E_p}{\sqrt{3}}$	E_p
Power per phase	—	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.I_d$	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.I_d.K$	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.I_d.K$	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.I_d.K$
Total power ..	$V_{do}.m.I_d$	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.I_d.m.p.$	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.p.I_d.m$	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.p.I_d.m$	$\frac{E_{so}\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.p.I_d.m$
	$=V_{do}.I_{dc}$	$=V_{do}.I_{dc}$	$=V_{do}.I_{dc}$	$=V_{do}.I_{dc}$	$=V_{do}.I_{dc}$

TABLE VI
EXAMPLE OF POWER BALANCE

$p = 3 \quad m = 4 \quad k = 4$

Quantity	Equation	Reference	Example
D.C. power output D.C. voltage Direct current	$P_{do} = V_{do}.I_{dc}$ V_{do} I_{dc}	— — —	5,000 kW 500 volts 10,000 amp
Current per group	$I_d = \frac{I_{dc}}{m}$	E_q " 1	2,500 amp
Secondary voltage	$E_{ac} = \frac{V_{do}.\pi}{\sqrt{2}.p.\sin\frac{\pi}{p}}$	E_q " 18	427 volts
R.M.S. fundamental component of secondary current	$I_{1ac} = \frac{I_d.\sqrt{2}}{\pi}.\sin\frac{\pi}{p}$	E_q " 4	975 amp
Total secondary power	$P_{ac} = m.p.I_{1ac}.E_{ac}$	—	5,000 kW
R.M.S. fundamental component of primary current	$I_{1p} = \frac{I_d.\sqrt{2}}{\pi}.\sin\frac{\pi}{p}.K.\frac{E_{ac}}{E_p}$	E_q " 16	152 amp
Total primary power	$P_{pdo} = 3.E_p.I_{1p}$	—	5,000 kW
R.M.S. fundamental component of line current	$I_{1l} = \sqrt{3}.I_{1p}$	—	263 amp
Total line power	$P_{1l} = 3.I_{1p}.\frac{E_p}{\sqrt{3}}$	—	5,000 kW

This expression is easily derived from simple theory.

Consider now the simple double-star connection given as connection R in Table IV which for each secondary phase has a current wave-form as given in Fig. 8, *i.e.* p is equal to 3. Equation 6 and Fig. 8 both show that the fundamental of current is in phase with the secondary voltage, and by the process of 'transmission' this will apply to every part of the A.C. circuit. Using the equations derived in this section, Table V can be compiled to show the flow of power through a rectifier equipment, with the result that the power input equals both the power output and the power transmitted in each circuit. This result is consistent with the initial assumptions of zero losses. A further interesting result arises from the fact that the fundamental component of current is everywhere in phase with its voltage, *i.e.* there is no reactive kVA demanded by the rectifier, which again is consistent with the assumption of zero reactance in the A.C. side. This in fact means operation at unity power factor.

The flow of power has been calculated as a result of Fourier analysis of a simple rectangular wave-form, followed by a study of the manner in which the constituent components will flow. In the next section allowance will be made for reactance in the A.C. side and a series of correction factors introduced to be applied to the results already obtained, but a simple practical example is first given to conclude this section.

Practical Example

Consider an equipment operating under the ideal conditions as laid down at the beginning of this section, with the following particulars:—

D.C. power output P_{do} = 5,000 kW.

D.C. voltage V_{do} .. = 500 volts.
Direct-current I_{dc} = 10,000 amp.
Connection T, Table IV = quadruple zig-zag, delta primary.
A.C. primary voltage E_p = 11,000 volts.

The important properties of the equipment can be calculated as in Table VI where reference is made to the appropriate equation in the text. The fundamental component of current is calculated for each part of the circuit and multiplied by its appropriate voltage to show that flow of power is the same throughout the circuit. Although this example is simple and the result self-evident from other considerations, it is given here as a basis for the more complicated sections to follow. The point to be noted is that power in each part of the circuit is calculated from the current component which was itself derived from the Fourier analysis of the secondary current without the necessity for analysis of the more complicated primary current wave-shapes.

The example given in Table VI can be used to illustrate the application of Tables II, III and IV in calculating the harmonic components. From Table IV connection T, the harmonics in the line current are given as a percentage of $I_d \cdot E_{so}/E_p$, i.e. $2{,}500 \cdot 427/11{,}000 = 97$.

The following results may therefore be tabulated.

Harmonic ..	1st	11th	13th	23rd	25th
I_n as % of $I_d \cdot E_{so}/E_p$	270	24.5	20.8	11.7	10.8
		(from Table IV)			
I_n amplitude	263	23.8	20.2	11.4	10.5
		(from $\%I_n \times 97$)			

The validity of the Laws of Harmonic Numbers and Harmonic Amplitudes (Equations 13 and 14) are evident from the values derived above.

This article will be concluded in Part Two to be published in the June 1954 *issue of this Journal, Volume* 13, *No.* 6.

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